Wind-Driven Ekman Transport of Curvilinear Flows

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Motivation

❖ Why ocean Ekman dynamics?

Ekman theory elucidates how ocean responds to wind forcing, leading to a horizontal mass transport called Ekman transport.

Spatial variability in the Ekman transport also generates vertical velocities in the near-surface ocean (Ekman pumping).

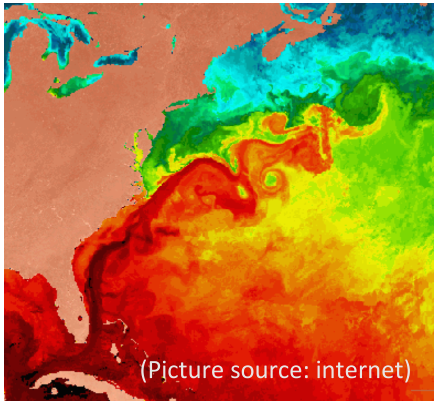
Ekman pumping provides a boundary condition for the interior flow, crucial to many theories of the general ocean circulation and biogeochemical processes.

❖ Why curvilinear flows?

Plane parallel flow is not a good approximation for realistic ocean flows.

Many important oceanic flows have $R_o \sim 1$, including western boundary currents, flows at low latitudes, and submesoscale currents and vortices.

The Gulf Stream



Introduction: Development of the Ekman Theory

People	Ekman (1905)	Stern (1965)	Wenegrat & Thomas (2017)
Content	Transport depends on the stress and the Coriolis parameter only.	Allows for shear in the surface velocity field to affect the transport: "nonlinear" Ekman theory.	Extends Stern's results to better account for curvature in the flow path.
Equations	$fu = \frac{\partial \tau_y}{\partial z}$ $-fv = \frac{\partial \tau_x}{\partial z}$	$(f+\zeta)u = \frac{\partial \tau_y}{\partial z}$ $-(f+\zeta)v = \frac{\partial \tau_x}{\partial z}$	$R_0 \bar{u} \frac{\partial v}{\partial s} + (1 + R_0 2\Omega)u = \frac{\partial \tau_n}{\partial z}$ $R_0 \bar{u} \frac{\partial u}{\partial s} - (1 + R_0 \zeta)v = \frac{\partial \tau_s}{\partial z}$
Ekman Transport	$U_{Ek} = \frac{\tau_y}{f}$ $V_{Ek} = -\frac{\tau_x}{f}$	$U_{Ek} = \frac{\tau_y}{f+\zeta}$ $V_{Ek} = -\frac{\tau_x}{f+\zeta}$	$R_0 \bar{u} \frac{\partial V_{Ek}}{\partial s} + (1 + R_0 2\Omega)U_{Ek} = \tau_n$ $R_0 \bar{u} \frac{\partial U_{Ek}}{\partial s} - (1 + R_0 \zeta)V_{Ek} = \tau_s$
Assumptions	Homogeneous deep ocean, steady flow, no pressure gradients, no horizontal friction…	Valid for plane parallel flows (e.g., straight jets); however, validity for curvilinear flows has been questioned by Wenegrat & Thomas.	Curvilinear flows, with $R_{oEk} \ll 1$ and $R_o < 1$.

Questions to ask

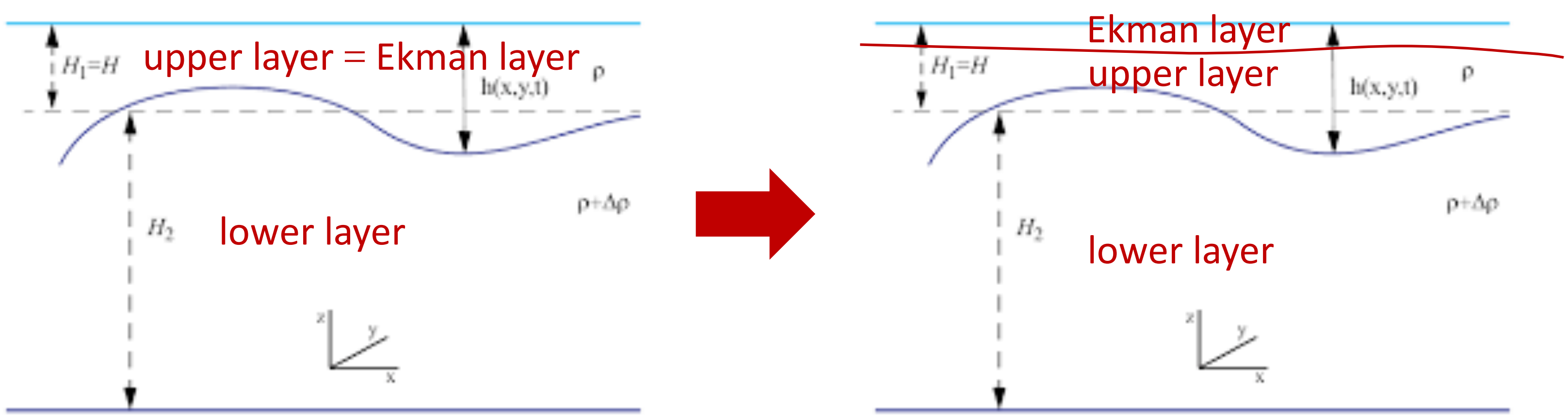
- 1. Ekman transport of curvilinear flows

  1.1 How is the Ekman transport of a background vortex different from intuition (i.e., from classic Ekman theory)?

  1.2 How can we apply the Wenegrat & Thomas result for curvilinear flows to complex (e.g., turbulent) flow fields ?

- 2. Ekman dynamics of a two-layer model

  2.1 Typically, wind stress is applied as a body force over the upper layer. But can we use explicit representation of the Ekman layer to force a two-layer model? If so, how do these different assumptions about the Ekman layer affect the results?



Model Framework for the Ekman Layer

❖ Ekman transport equations in our model

$$\frac{\partial U_{Ek}}{\partial t} = -\frac{\partial B}{\partial x} + (f + \zeta_0)V_{Ek} + \zeta_{Ek} v_0 + \tau_x - Ah\nabla^4 U_{Ek}$$

$$\frac{\partial V_{Ek}}{\partial t} = -\frac{\partial B}{\partial y} - (f + \zeta_0)U_{Ek} - \zeta_{Ek} u_0 + \tau_y - Ah\nabla^4 V_{Ek}$$

unsteady term; Bernoulli term; vorticity term; wind forcing term; diffusion term

Notice:

1. u_0, v_0, ζ_0 represent the balanced curvilinear flow.

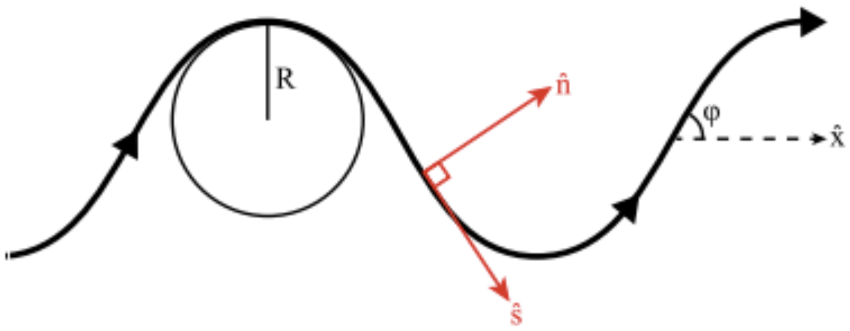
2. $B = \frac{1}{2}(U_{Ek}u_0 + V_{Ek}v_0)$

3. Units: U_{Ek} $(\mathrm{m^2s^{-1}})$, u_0 $(\mathrm{ms^{-1}})$

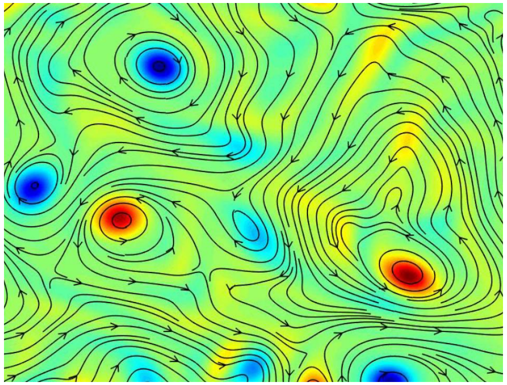
Model Framework for the Ekman Layer

- We avoid using curvilinear coordinates compared with Wenegrat & Thomas.

Curvilinear Coordinates



Turbulence Streamlines



- Note that all of the formulations for the Ekman layer assume "pressureless dynamics". That is, the HPGF affects the interior flow but not the Ekman correction to this flow.

1.1 How is the Ekman transport of a balanced vortex different from intuition?

Fig.1 Our Model Simulations

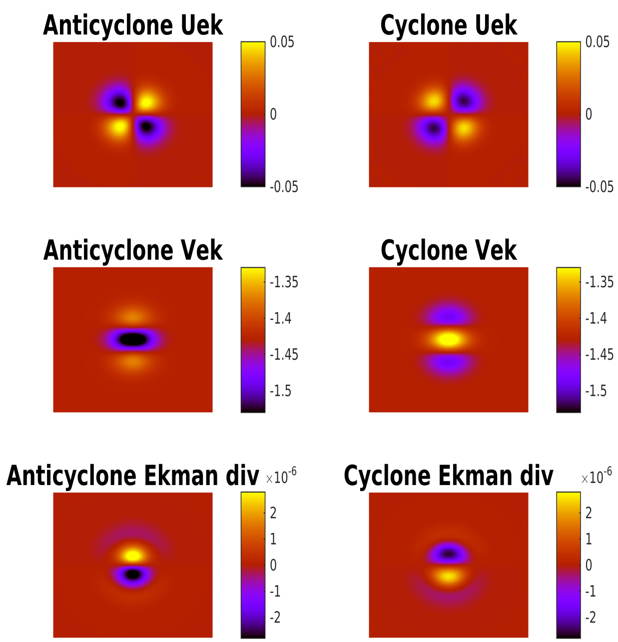
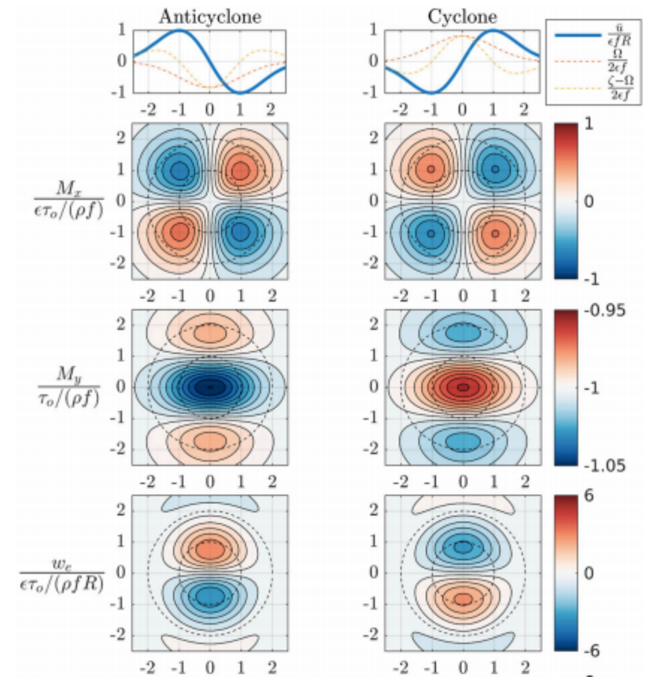


Fig.2 Wenegrat & Thomas simulations

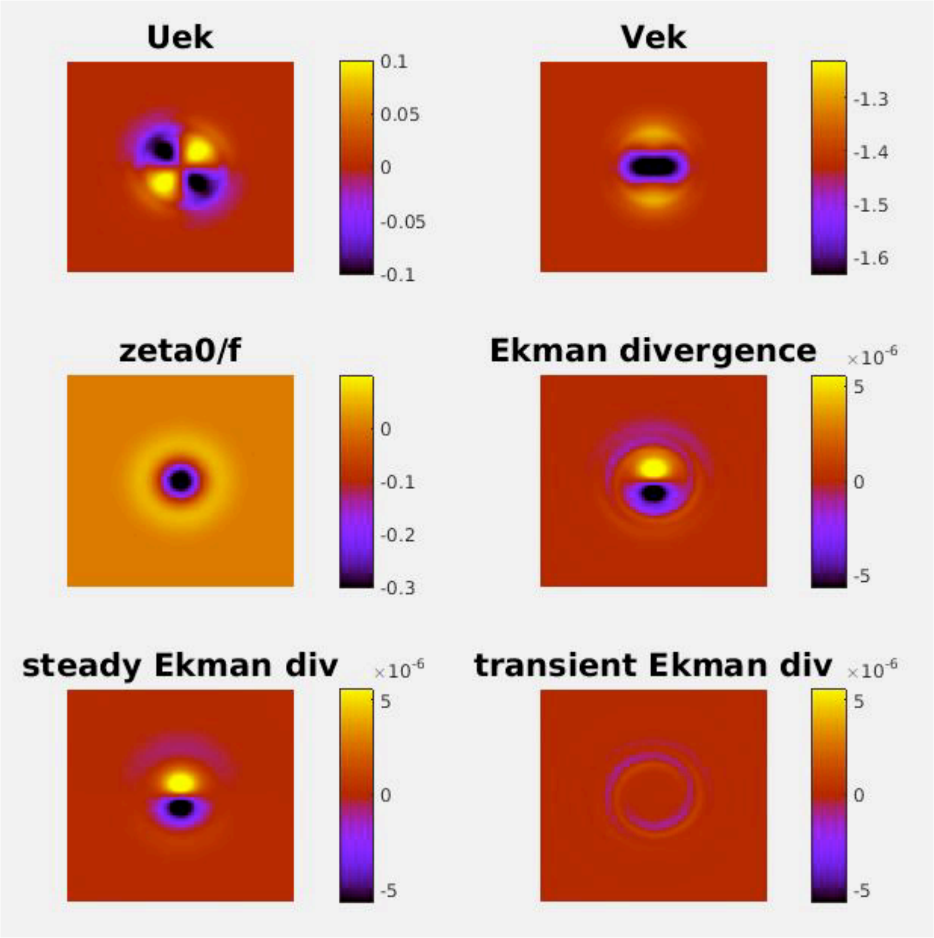


The zonal transport develops a quadrupole pattern, emphasizing that the nonlinear Ekman transport is not strictly perpendicular to the wind stress.

The meridional transport converges (diverges) on the north (south) side of the cyclonic vortex, with the pattern reversed for the vortex with anticyclonic flow.

1.2 But our model produces transients, whereas the Wenegrat & Thomas model does not.

Film.1 An Example (Anticyclone) of Transients

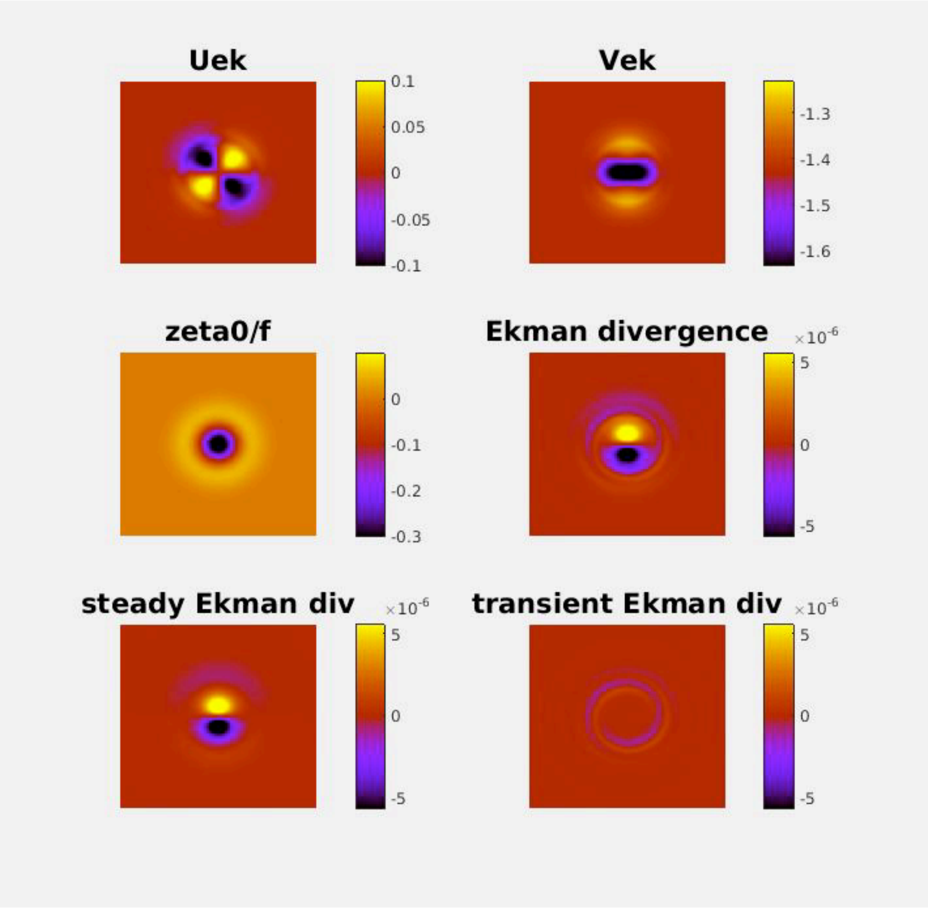


Three sources of transients:

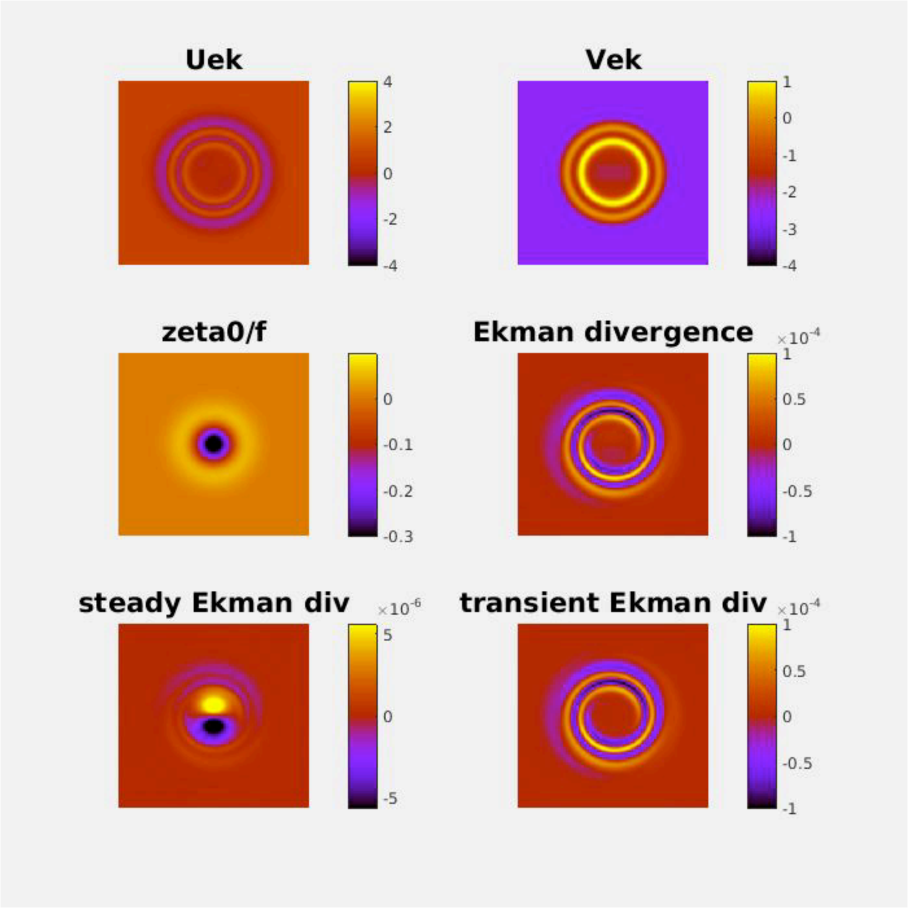
1. With or without ramp (i.e., to slowly switch on the wind forcing)
2. Numeric factors
3. Cyclones vs Anticyclones

1.2.1 Sources of Transients: with and without Ramp

Film.2 With Ramp (n=10)



Film.3 Without Ramp



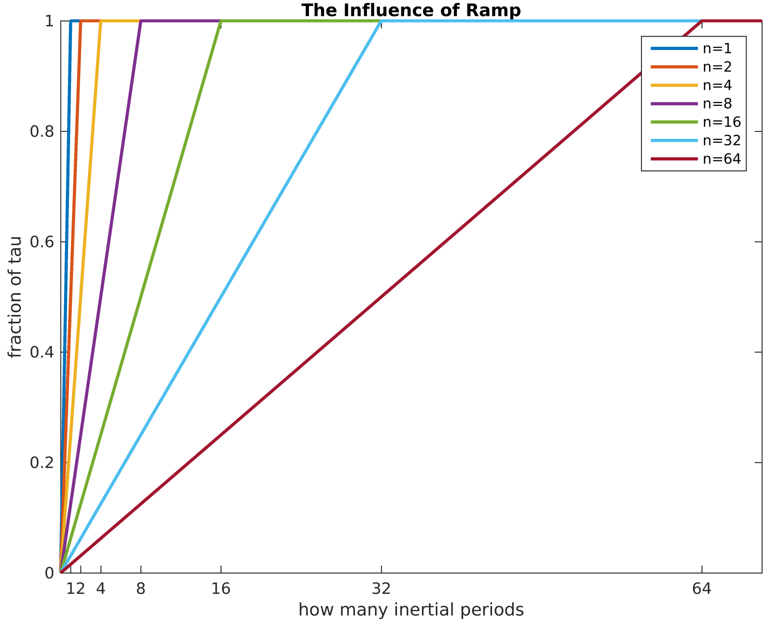
Follow divergence fields: slowly turning on wind stress reduces transients, whereas abruptly applying wind forcing produces dominant transients.

1.2.1 Sources of Transients: with and without Ramp

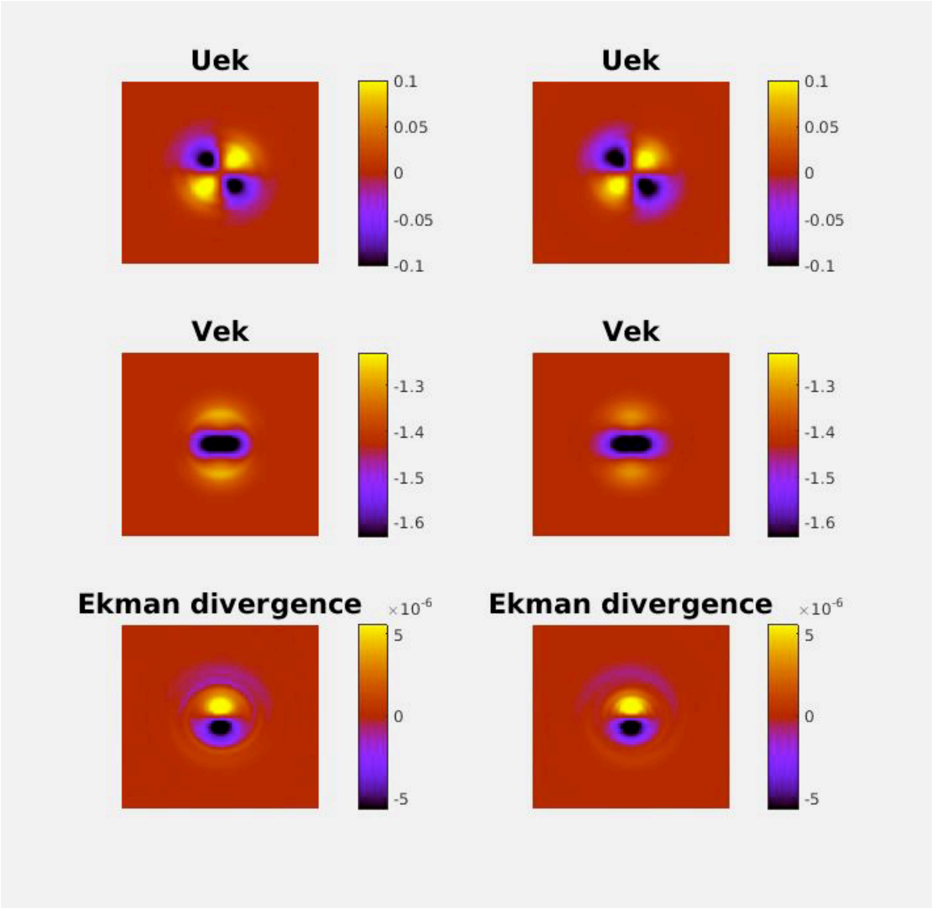
Inertial period = $\frac{2\pi}{f} \sim$ 1 day

n = using how many inertial periods to turn on the wind stress entirely

Fig.3 The Switch-on of Ramp



Film.4 Ramp (n=1) vs Ramp (n=64)



Even wind stress is turned on slowly, we could not get rid of transients.

Recap

- 1. Ekman transport of curvilinear flows

1.1 How is the Ekman transport of a balanced vortex different from intuition?

Transport can include a component that is not perpendicular to the stress.
Transport can include high frequency transients that are easily excited when wind stress changes abruptly.

1.2 How can we apply the Wenegrat & Thomas result for curvilinear flows to complex (e.g., turbulent) flow fields ?

Explicitly including time dependence eliminates the need to integrate along curvilinear streamlines.

Next, we consider resolution dependence (numeric factors). We find that at low resolution spurious waves can be excited both in our model and (seemingly) in the MITgcm.

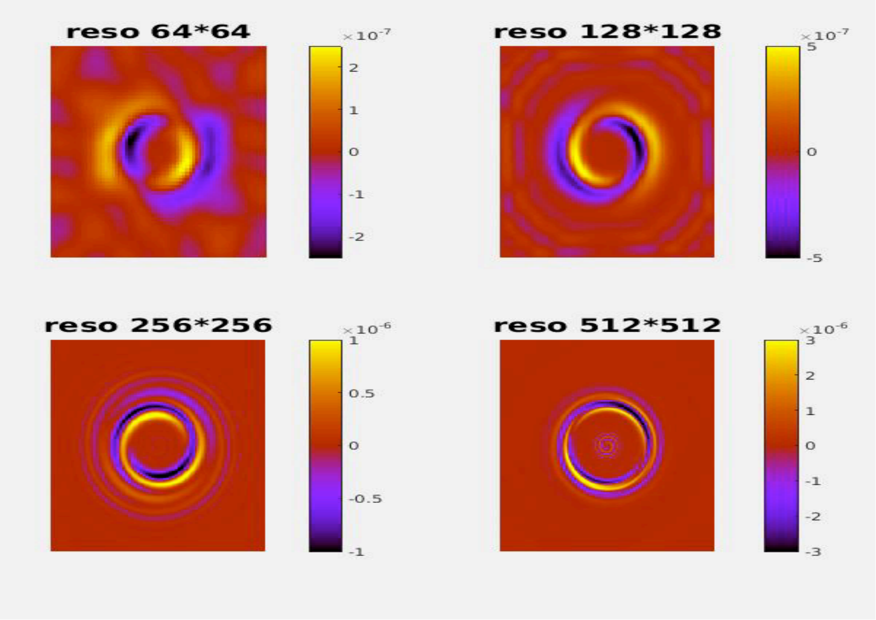
1.2.2 Sources of Transients: Numeric factors

Note that far from the vortex, $u_0 = v_0 = 0$ and the Ekman equations reduce to

$$\frac{\partial U_{Ek}}{\partial t} = fV_{Ek} + \tau_x - Ah\nabla^4 U_{Ek}$$

$$\frac{\partial V_{Ek}}{\partial t} = -fU_{Ek} + \tau_y - Ah\nabla^4 V_{Ek}$$

Film.5 Transients Associated with Resolution



Since there is no x-y dependence in the coefficients, there should be no propagation. However, we do see propagation away from the vortex (especially at low resolution). This appears to be a numerical artefact. A similar problem also seems to exist in the MITgcm.

1.2.2 Sources of Transients: Numeric factors

Fig.4 MITgcm - Stern

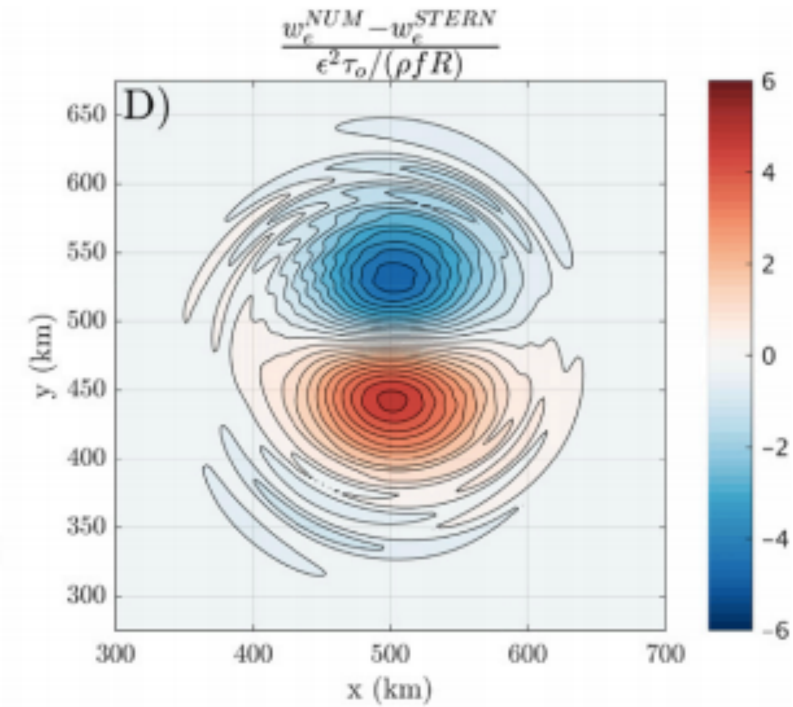
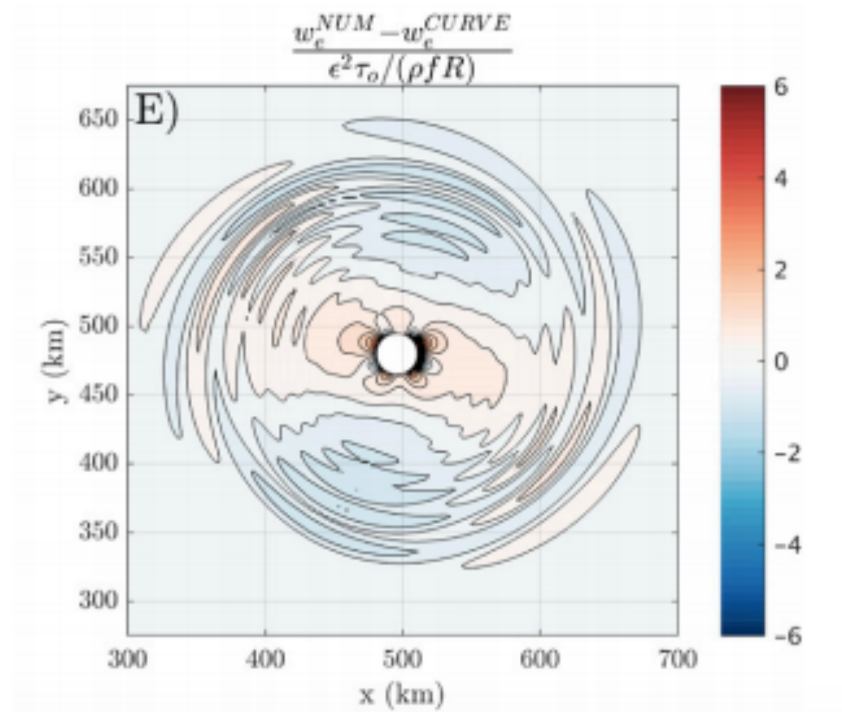


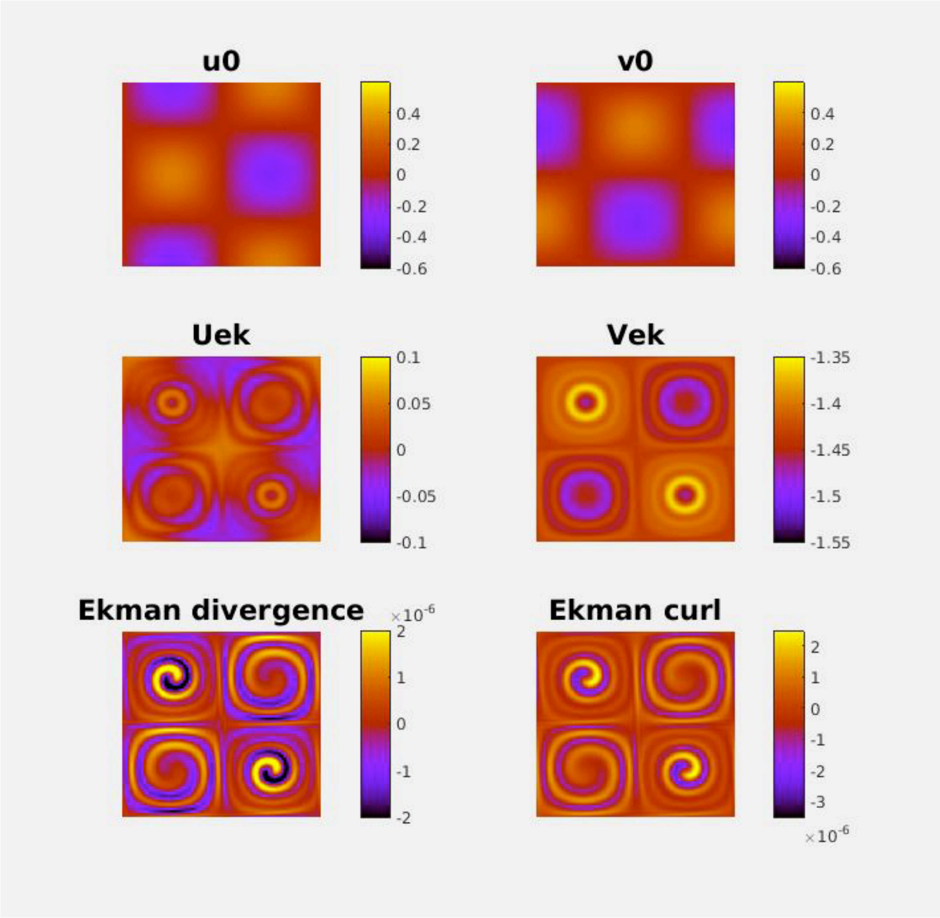
Fig.5 MITgcm - Wenegrat



1. Ekman pumping calculated from Wenegrat & Thomas formulation is more accurate than the one calculated from Stern formulation.
2. We also see propagation away from the vortex and this problem may exist in the MITgcm, since both Stern vortex model and Wenegrat vortex model are analytical.

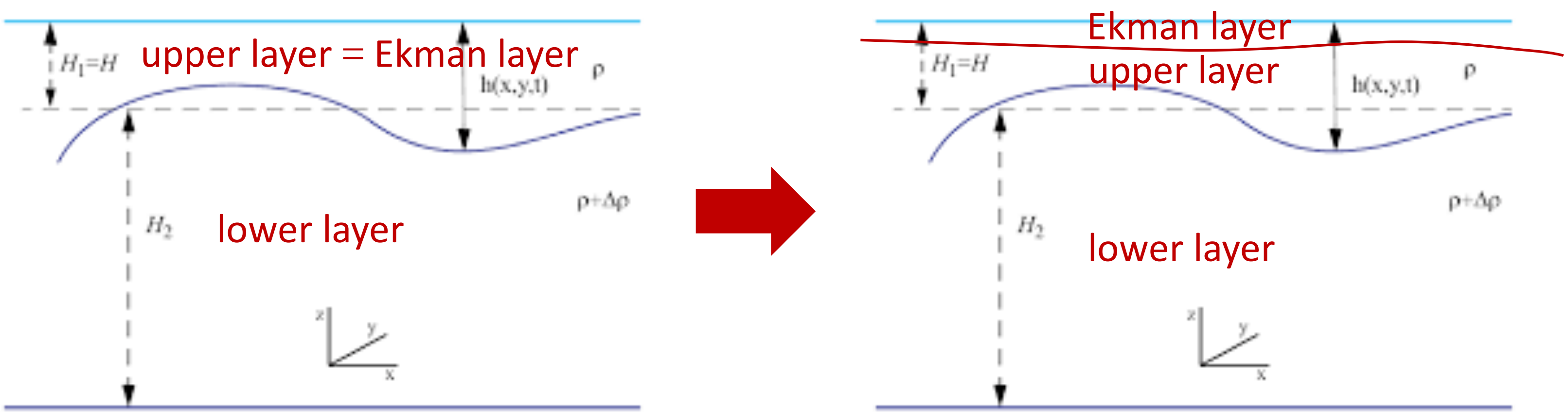
1.2.3 Sources of Transients: Cyclones vs Anticyclones

Film.6 Four-Eddy Case



2.1 Do these different models of the Ekman layer lead to different solutions for the interior flow?

- Typically, wind forcing is treated as a body force over the top layer. However, we consider a two-layer shallow water model with a sub Ekman layer in the top layer.

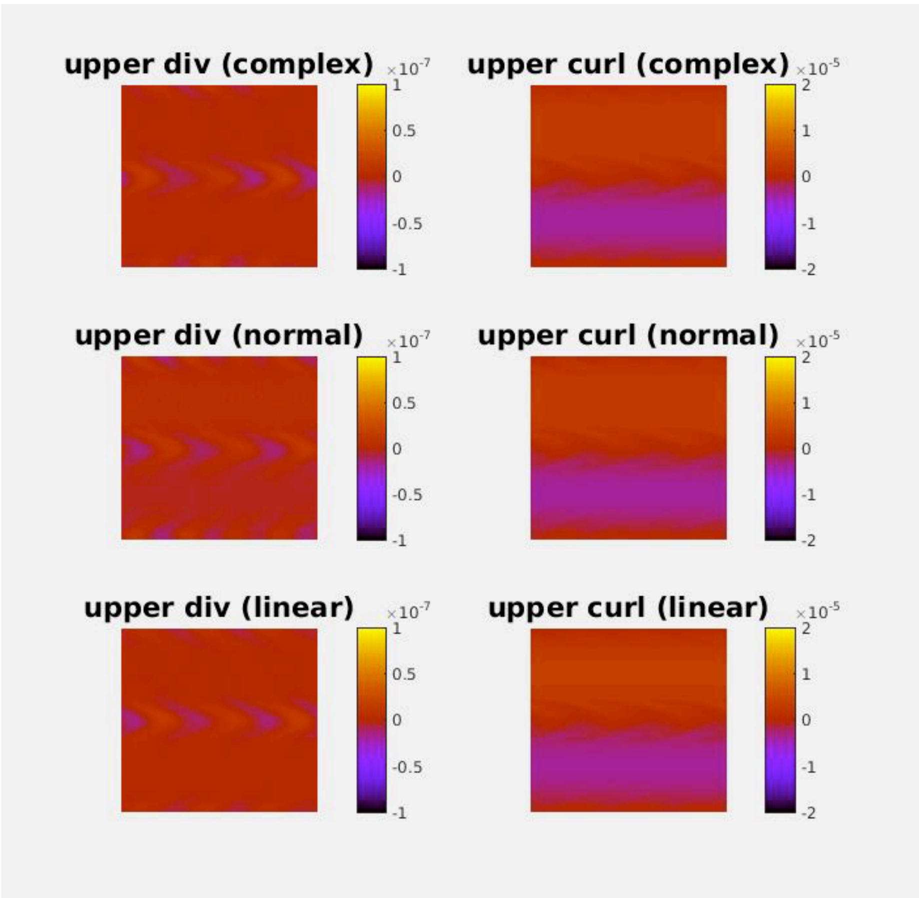


- Thus, we can use div(Uek) as a forcing in the upper layer mass equation.

- Model setup: two-layer rigid lid, domain size (1000km*1000km), resolution (256 grid points*256 grid points), Lrossby=28.6km, wind forcing taux is a periodic cosine function of y but does not change with time or x.

2.1 Do these different models of the Ekman layer lead to different solutions for the interior flow?

Film.7 Comparison of 3 Versions of Model



Compare three versions:

i) Normal

$$\frac{\partial u1}{\partial t} = -\frac{\partial B}{\partial x} + (f + \zeta)v1 + \frac{\tau_x}{H1 - \eta} \dots$$

ii) Linear

$$\frac{\partial \eta}{\partial t} = -div\big(u1(H1 - \eta)\big) - div(U_{ek})$$

$$V_{Ek} = -\frac{\tau_x}{f}$$

iii) Complex

Like ii), but with time dependent U_{Ek} as described previously

Future work

- ❖ Further tests on model resolution to better quantify spurious generation of transients

- ❖ More thorough comparison of the three different model formulations

  i) different forcing strengths (Rossby numbers)
  ii) spectral analysis
  iii) time dependent forcing

We are interested in whether and how the different forcing types affect both the geostrophic (e.g., slowly varying) part of the flow and how it might affect high frequency transients, such as Poincaré and near-inertial waves.

- ❖ With our modified forcing scheme, a spatially constant Ekman transport does not influence the interior velocity (i.e., because it produces no Ekman pumping). Either need to add walls or need to consider the area-averaged momentum budget more carefully.

Thank you for your attention.