Flow-dependent Ekman Theory

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Introduction: classic Ekman model

$$f\hat{k}\times\vec{u} = \frac{1}{\rho_0}\nabla p + A_z\frac{\partial^2\vec{u}}{\partial z^2}$$

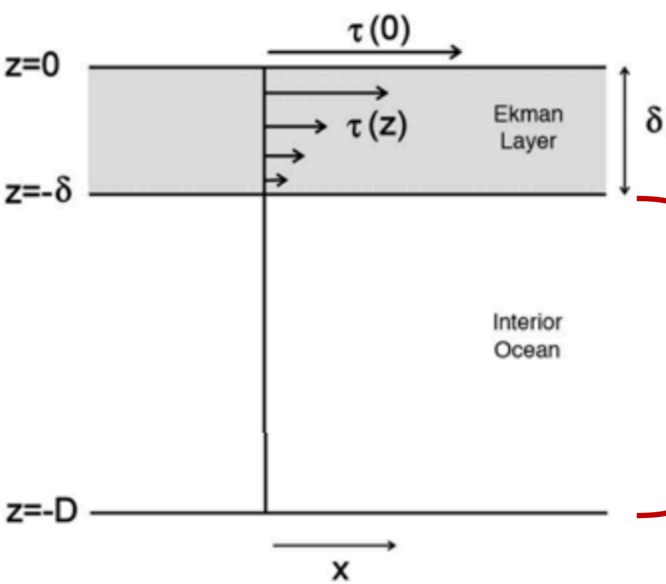
$$\vec{u} = \vec{u}_g + \vec{u}_{Ek}$$

$$f\hat{k}\times(\vec{u}_g + \vec{u}_{Ek}) = \frac{1}{\rho_0}\nabla p + A_z\frac{\partial^2(\vec{u}_g)}{\partial z^2} + A_z\frac{\partial^2(\vec{u}_{Ek})}{\partial z^2}$$

0

geostrophic balance

Ekman balance



Ekman balance ➡ How to improve?

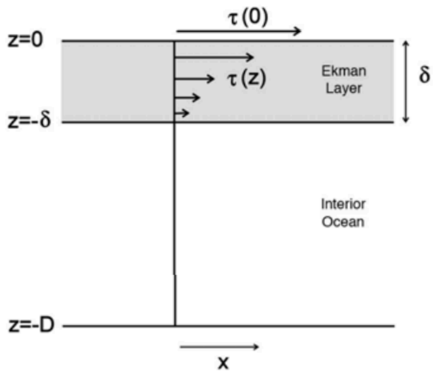
Interior balance $\stackrel{?}{=}$ geostrophic balance

Introduction: development of the Ekman theory

People	Ekman (1905)	Stern and Niiler (1960s)	Wenegrat & Thomas (2017)
Content	Transport depends on the stress and the Coriolis parameter only.	Allows for shear in the surface velocity field to affect the transport: "nonlinear" Ekman theory.	Extends early results to better account for curvature in the surface flow path.
Ekman Transport	$U_{Ek} = \frac{\tau_y}{f}$ $V_{Ek} = -\frac{\tau_x}{f}$	$U_{Ek} = \frac{\tau_y}{f+\zeta}$ $V_{Ek} = -\frac{\tau_x}{f+\zeta}$	$R_0 \bar{u} \frac{\partial V_{Ek}}{\partial s} + (1 + R_0 2\Omega) U_{Ek} = \tau_n$ $R_0 \bar{u} \frac{\partial U_{Ek}}{\partial s} - (1 + R_0 \zeta) V_{Ek} = \tau_s$
Assumptions	Homogeneous, infinitely deep ocean.	Valid for plane parallel flows (e.g., straight jets); however, not explicitly solved for flows with curvature.	Curvilinear flows, with $R_{oEk} \ll 1$ and $R_o < 1$.

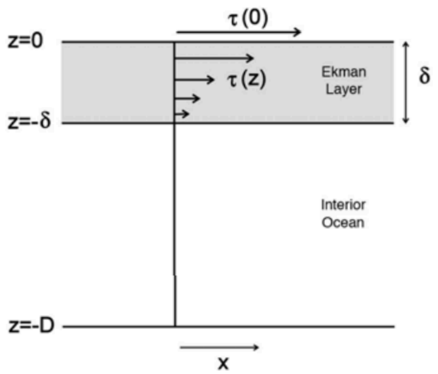
Questions:

- What is the impact of various Ekman formulations on the Ekman-layer transport for a fixed wind and a fixed oceanic balanced vortex?



Constant wind stress at the surface.
Constant vortex in the ocean interior.

- What is the impact of various Ekman formulations on the interior flow when the Ekman-layer is coupled to the interior?



Seek solutions for the interaction between Ekman dynamics and the interior, with different wind stress (constant or time-dependent).

Model framework for the Ekman layer

❖ Ekman transport equations in our model

$$\frac{\partial U_{Ek}}{\partial t} = -\frac{\partial B}{\partial x} + (f + \zeta_0)V_{Ek} + \zeta_{Ek}v_0 + \tau_x - A_h\nabla^4 U_{Ek}$$

$$\frac{\partial V_{Ek}}{\partial t} = -\frac{\partial B}{\partial y} - (f + \zeta_0)U_{Ek} - \zeta_{Ek}u_0 + \tau_y - A_h\nabla^4 V_{Ek}$$

unsteady term; Bernoulli term; vorticity term; wind forcing term; diffusion term

where

1. u_0, v_0, ζ_0 represent the balanced curvilinear flow.

2. $B = \frac{1}{2}(U_{Ek}u_0 + V_{Ek}v_0)$

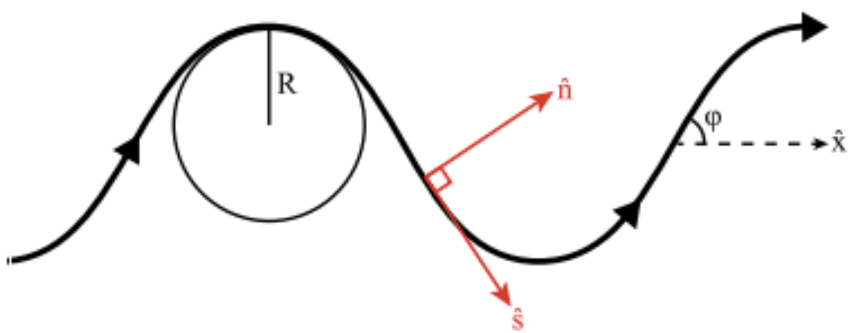
3. Units: U_{Ek} $(\mathrm{m^2 s^{-1}})$, u_0 $(\mathrm{ms^{-1}})$

4. The nonlinear Ekman self-advection terms are neglected.

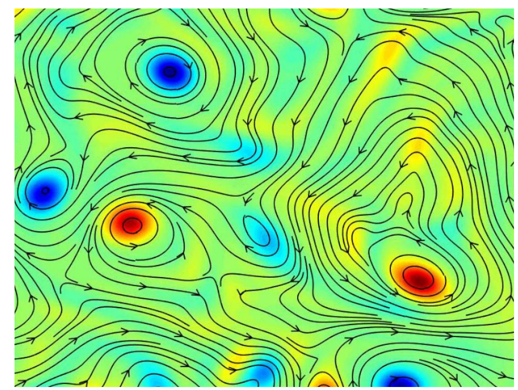
Model framework for the Ekman layer

- We extend Wenegrat & Thomas formulation by adding a time-dependent term. This step removes the need for integrating along streamlines.

Curvilinear Coordinates



Turbulence Streamlines



- Note that all of the formulations for the Ekman layer assume “pressureless dynamics”. That is, the HPGF affects the interior flow but not the Ekman correction to this flow.

How does curvature of a balanced vortex influence the Ekman dynamics?

Fig.1 Our Model Simulations

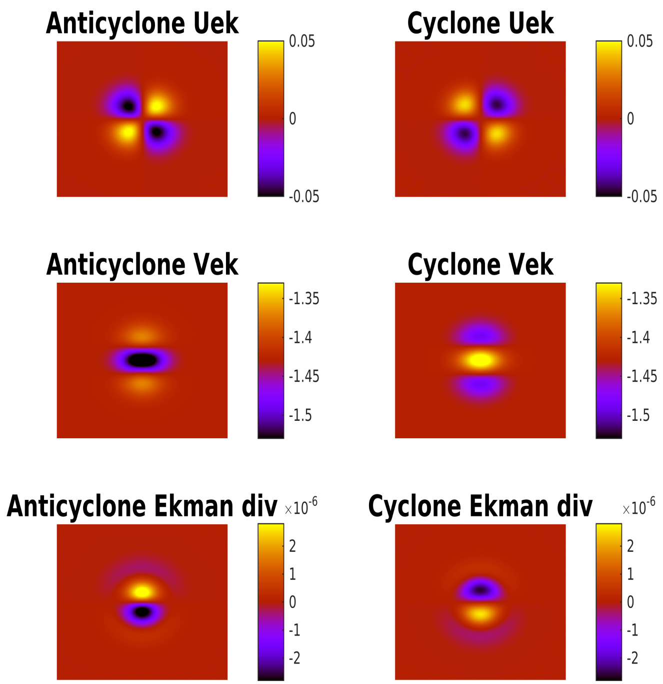
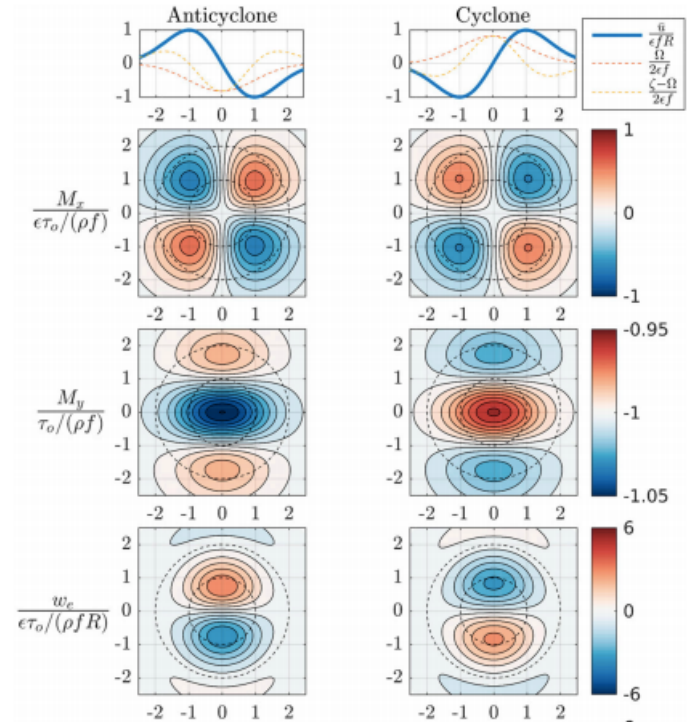


Fig.2 Wenegrat & Thomas simulations

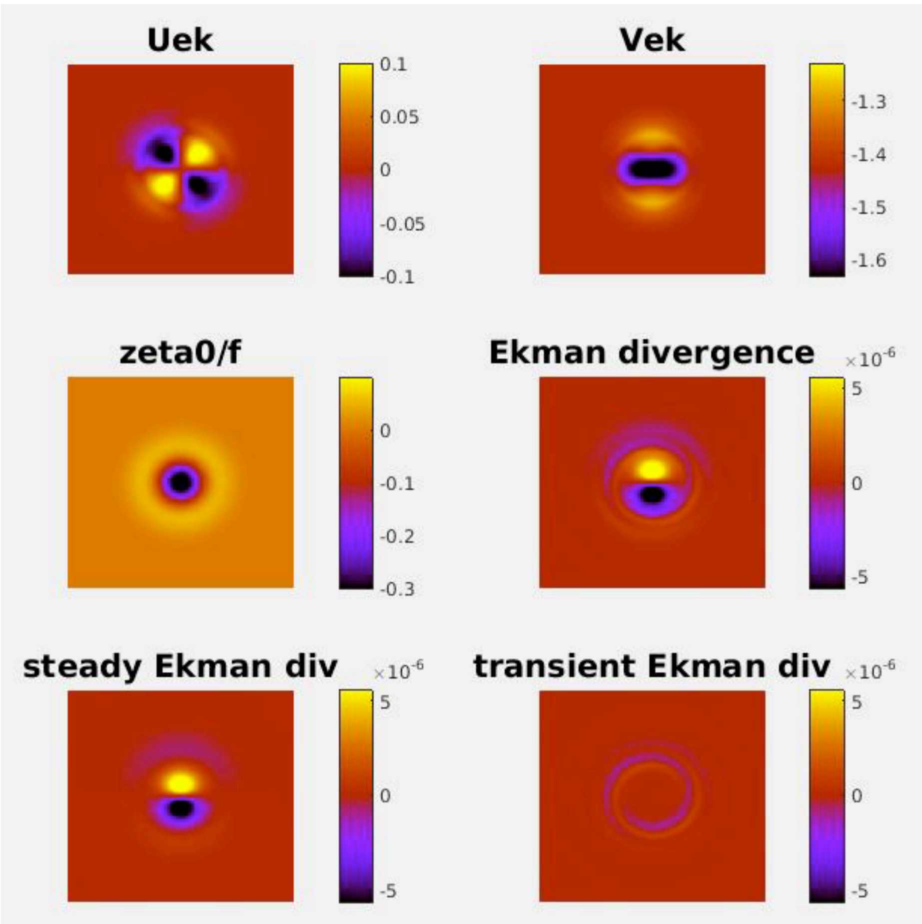


The zonal transport develops a quadrupole pattern, emphasizing that the nonlinear Ekman transport is not strictly perpendicular to the wind stress.

The meridional transport converges (diverges) on the north (south) side of the cyclonic vortex, with the pattern reversed for the vortex with anticyclonic flow.

How does curvature of a balanced vortex influence the Ekman dynamics?

Film.1 Our model simulation (An Anticyclone)



Our model produces transients, whereas the Wenegrat & Thomas model does not.

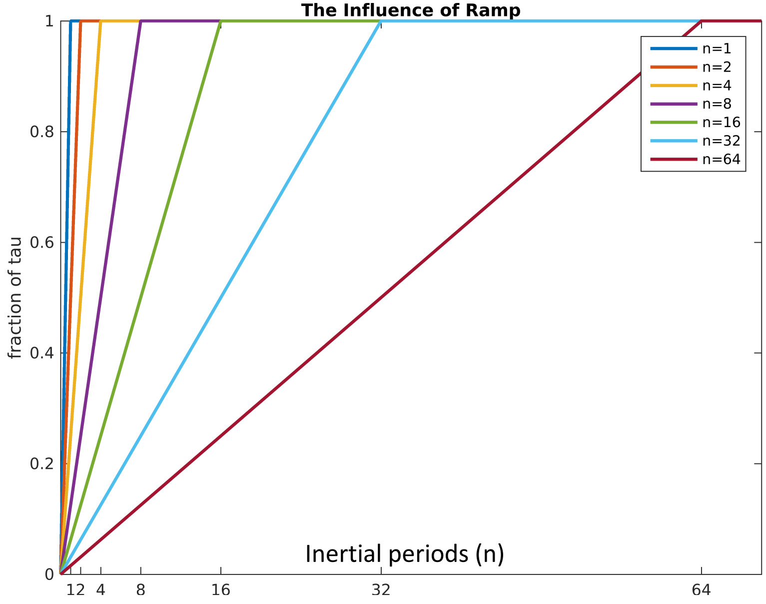
The main source of transients:
Swiftness of the wind increase from a rest state.

The source of transients: swiftness of the wind increase from a rest state.

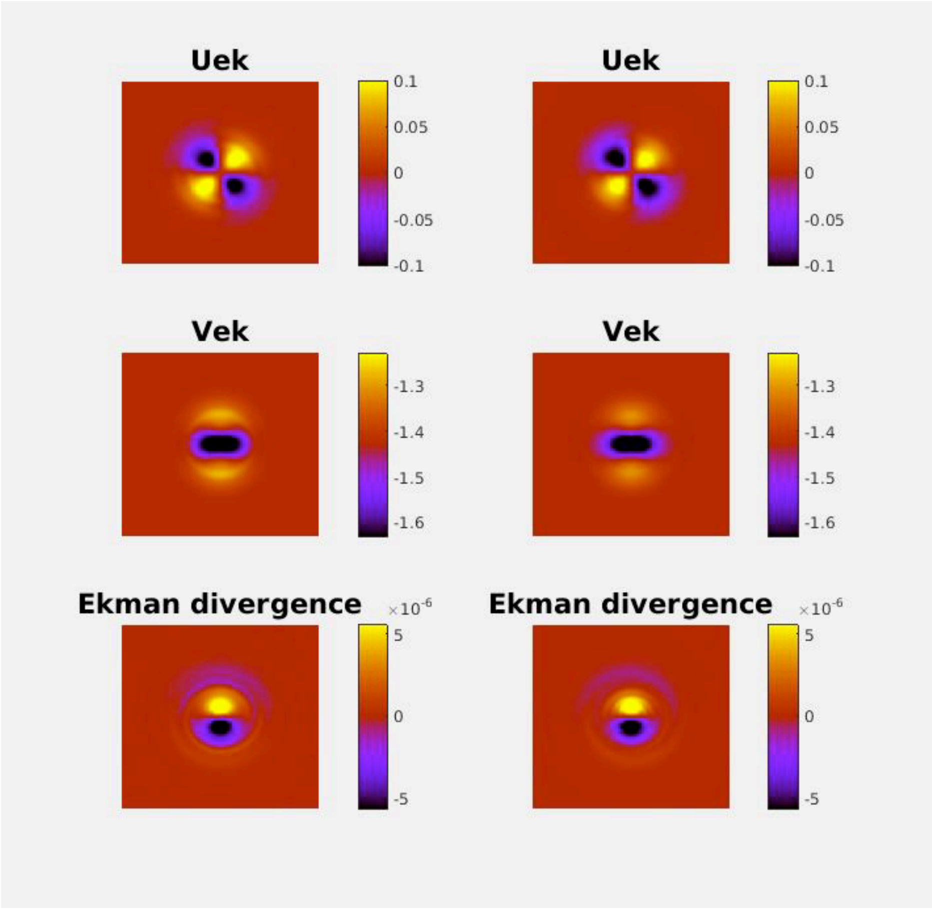
Inertial period = $\frac{2\pi}{f} \sim$ 1 day

n = number of inertial periods for which the wind stress is increased linearly

Fig.3 The Switch-on of Ramp



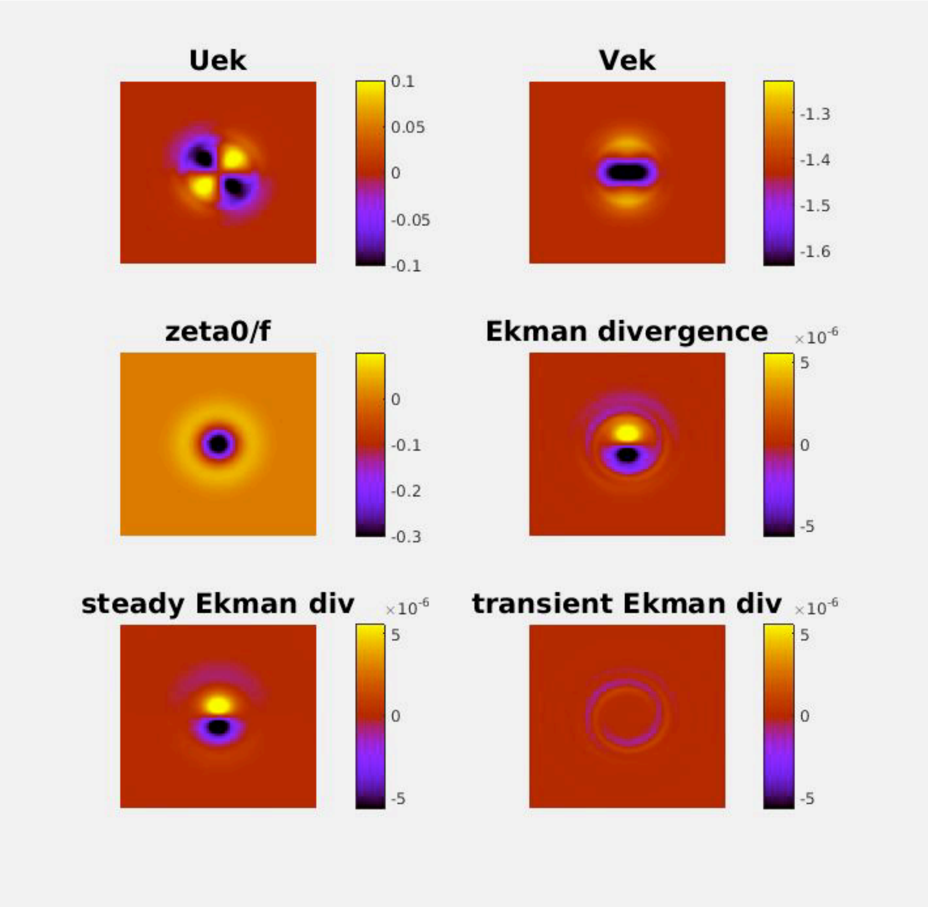
Film.4 Ramp (n=1) vs Ramp (n=64)



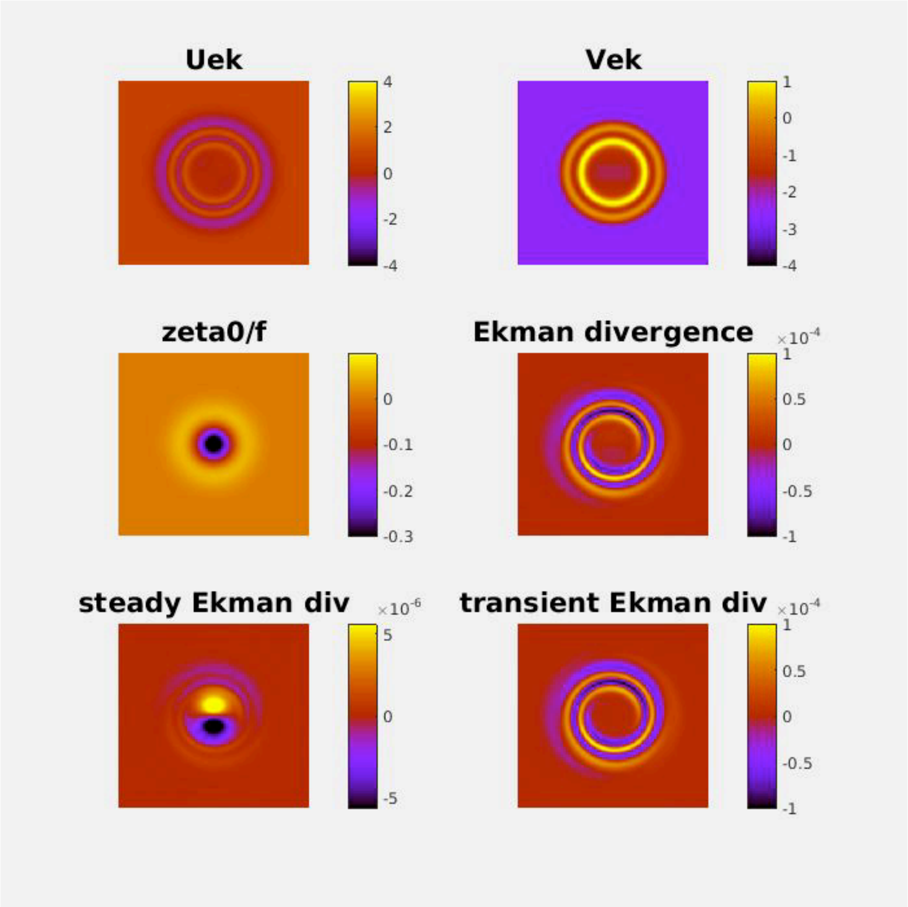
Transients are robust to very slow increase in wind stress.

The source of transients: swiftness of the wind increase from a rest state.

Film.2 With Ramp (n=10)



Film.3 Without Ramp



Focus on Ekman divergence: slowly turning on wind stress reduces transients, whereas abruptly applying wind forcing produces enhanced transients.

Recap

- What is the impact of various Ekman formulations on the Ekman-layer transport for a fixed wind and a fixed oceanic balanced vortex?

  Transport can include a component that is not perpendicular to the stress.
  Transport can include high frequency transients that are easily excited when wind stress changes abruptly.

- Application to a coupled Ekman-interior flow:

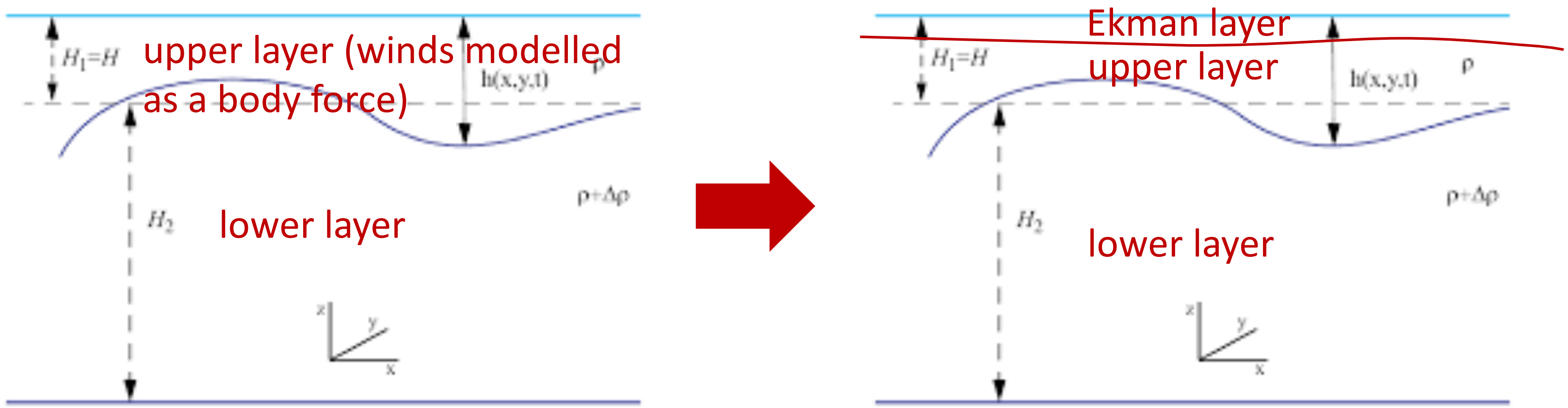
  Allowing for time dependent Ekman velocities eliminates the need to integrate along curvilinear streamlines.
  The time dependence can introduce a near-inertial (high-frequency) component to the Ekman pumping.

What is the impact of various Ekman formulations on the interior flow when the Ekman-layer is coupled to the interior?

We compare two formulations for the Ekman layer:

1. Wind stress is applied as a body force in the momentum equation
2. Use an explicit representation of the Ekman layer to force the mass equation

Standard method New method



We consider a two-layer shallow water model with a sub Ekman layer in the top layer. Thus, we can use "Ekman pumping" as a forcing in the upper layer mass equation.

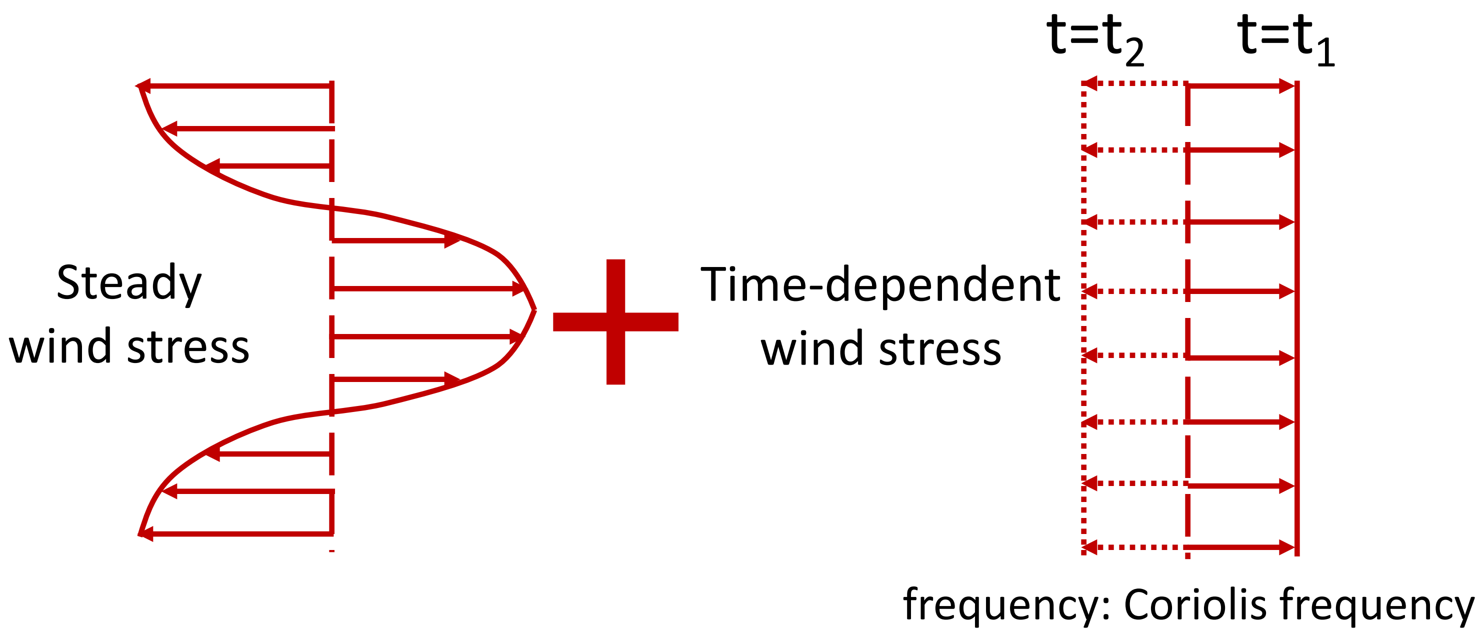
Model setup: two-layer rigid lid, domain size (1000km * 1000km), resolution (512 grid points * 512 grid points), wind forcing taux is a cosine function of y.

What is the impact of various Ekman formulations on the interior flow when the Ekman-layer is coupled to the interior?

We compare two formulations.

Simulations		Standard method	New method
Processes		Wind forcing → upper layer	Wind forcing → modified Ekman layer → upper layer
Equations	Ekman transport		$\frac{\partial}{\partial t}\vec{U}_E + (\vec{u}_1 \cdot \nabla)\vec{U}_E + \left(\vec{U}_E \cdot \nabla\right)\vec{u}_1 + f\hat{z}\times\vec{U}_E = \vec{\tau} - A_h\nabla^4\vec{U}_E$
	Upper-layer momentum	$\frac{\partial}{\partial t}\vec{u}_1 + (\vec{u}_1 \cdot \nabla)\vec{u}_1 + f\hat{z}\times\vec{u}_1 = \frac{\vec{\tau}}{h_1} - A_h\nabla^4\vec{u}_1$	$\frac{\partial}{\partial t}\vec{u}_1 + (\vec{u}_1 \cdot \nabla)\vec{u}_1 + f\hat{z}\times\vec{u}_1 = -A_h\nabla^4\vec{u}_1$
	Upper-layer mass	$\frac{\partial}{\partial t}h_1 + \nabla \cdot (h_1\vec{u}_1) = 0$	$\frac{\partial}{\partial t}h_1 + \nabla \cdot (h_1\vec{u}_1) = -w_E$ $(w_E = \nabla \cdot \left(\vec{U}_E\right))$

Wind forcing



We compare four cases:

Standard method	×	Steady wind stress
New method		Steady wind stress + time-dependent wind stress

Decomposition of QG and AG

Additionally, we are interested in whether the different forcing types affect independently the quasigeostrophic (i.e., slowly varying) part of the flow and the ageostrophic (fast) part of the flow, such as Poincaré and near-inertial waves.

For new method case, the forcing for interior flow is

$$w_E = QG\ part + AG\ part$$

QG part: $\overline{w_E} = \nabla \cdot (\frac{\hat{z} \times \vec{\tau}}{f})$

AG part: $w_E' = w_E - \overline{w_E}$

Decomposition of QG and AG

For standard case, the forcing for interior flow is

$$\nabla \times \frac{\vec{\tau}}{h} = QG\ part + AG\ part$$

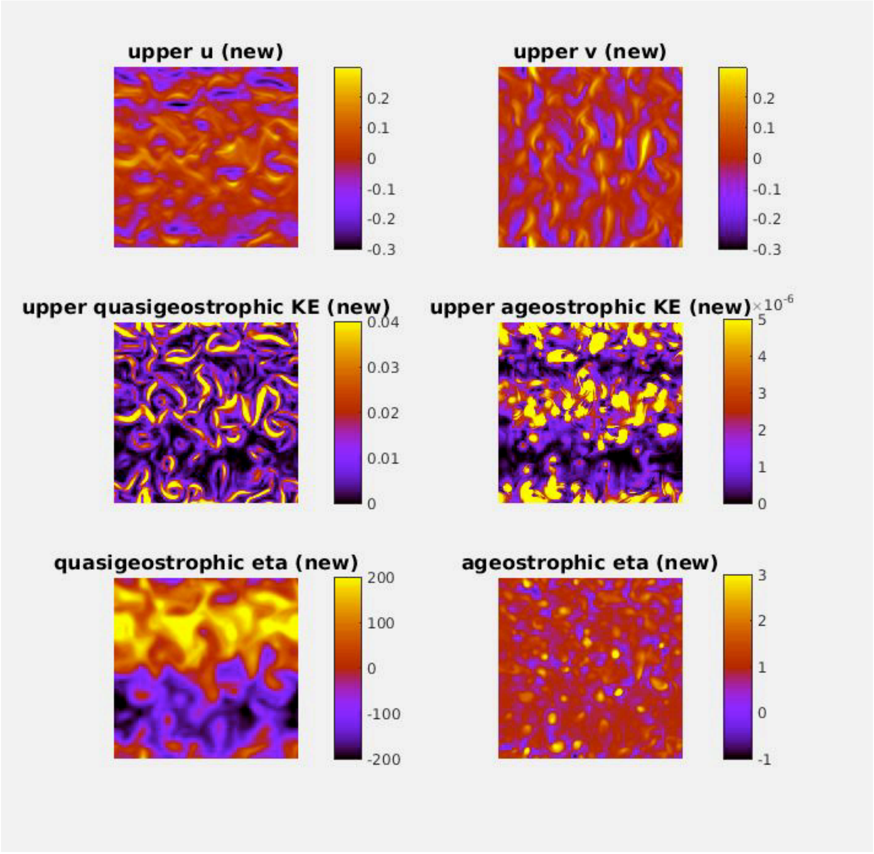
QG part: $\nabla \times \frac{\vec{\tau}}{H}$

AG part: $\nabla \times (\frac{\vec{\tau}}{h} - \frac{\vec{\tau}}{H})$

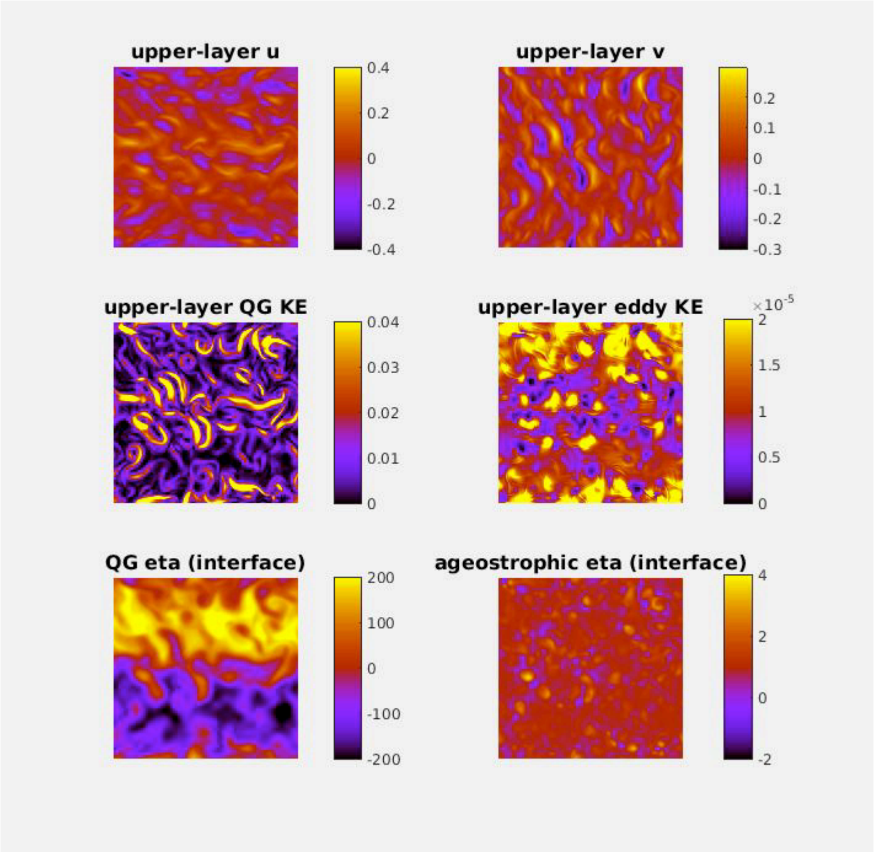
Notice that we can project the shallow water solution onto a QG part and an ageostrophic part. Thus, we can compare the QG part and AG part of energy output.

Comparison between steady and unsteady wind stress using the standard method

Film.5 Steady standard method

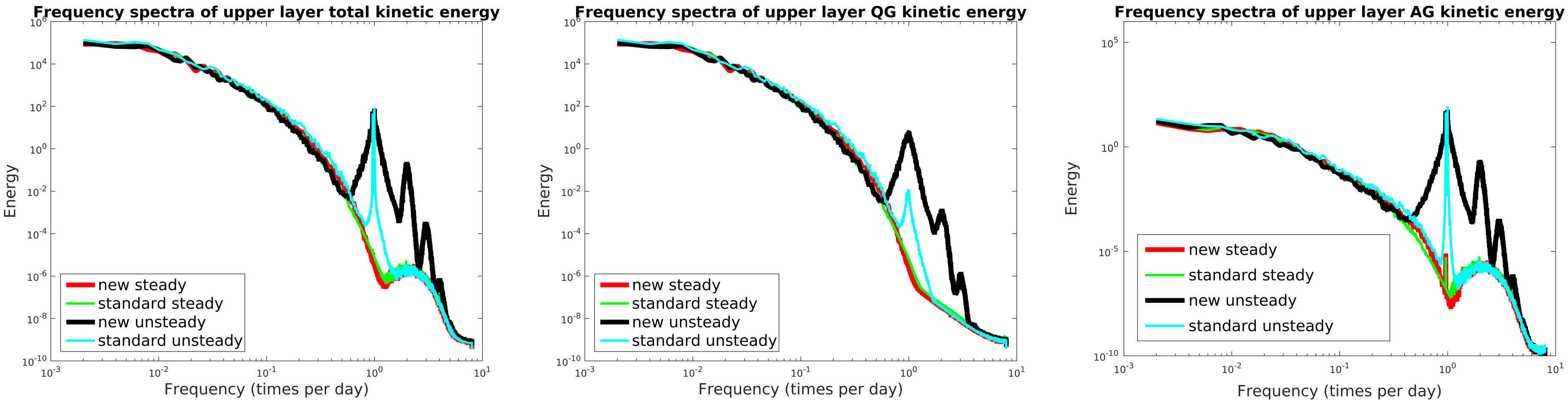


Film.6 Unsteady standard method



Comparison between steady and unsteady wind stress using the standard method

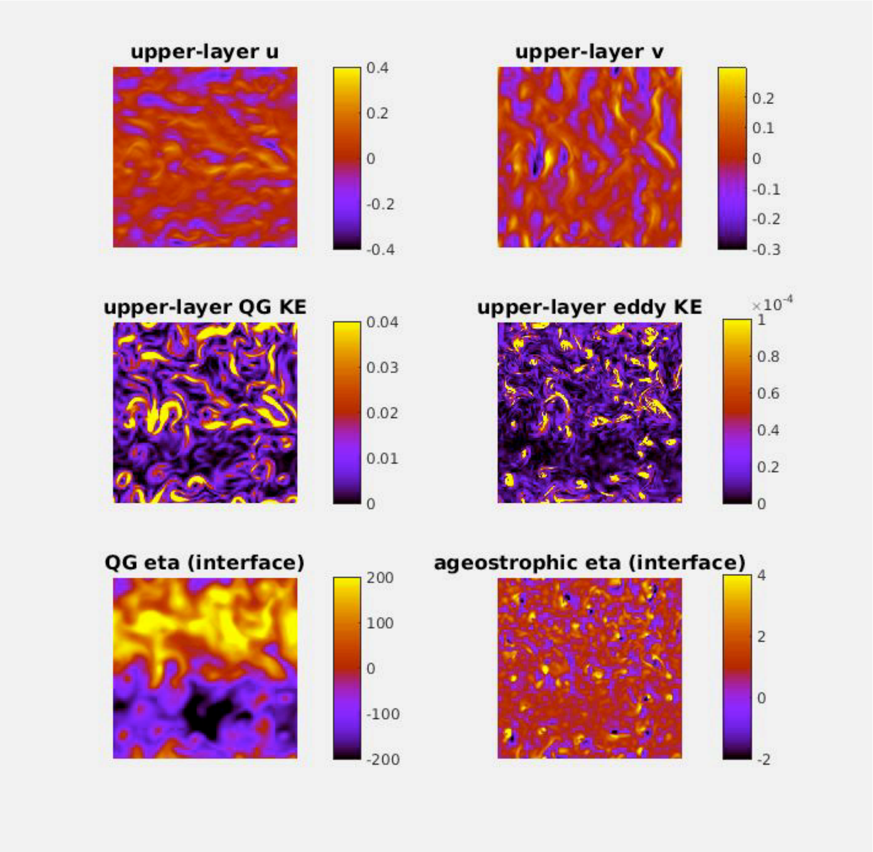
Fig.4 Upper-layer kinetic energy



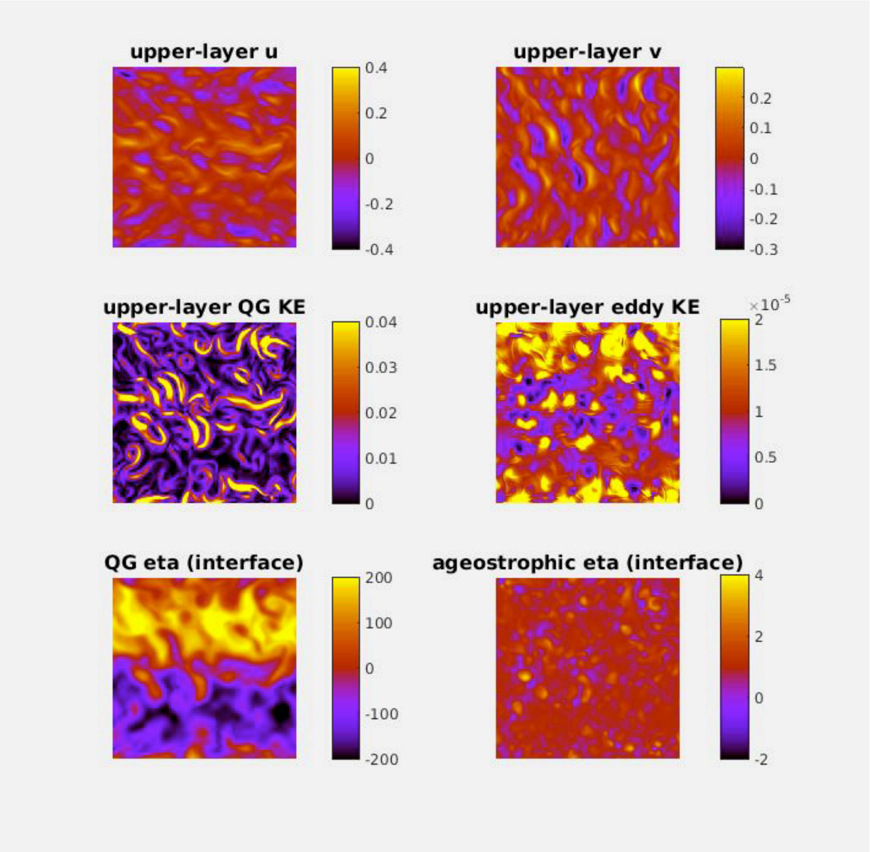
The high-frequency forcing excites near-inertial motion, and also the low-frequency nearly geostrophic part of the flow, similar to previous results from Taylor and Straub (2016).

Comparison of different forcing formulations

Film.7 Unsteady new method

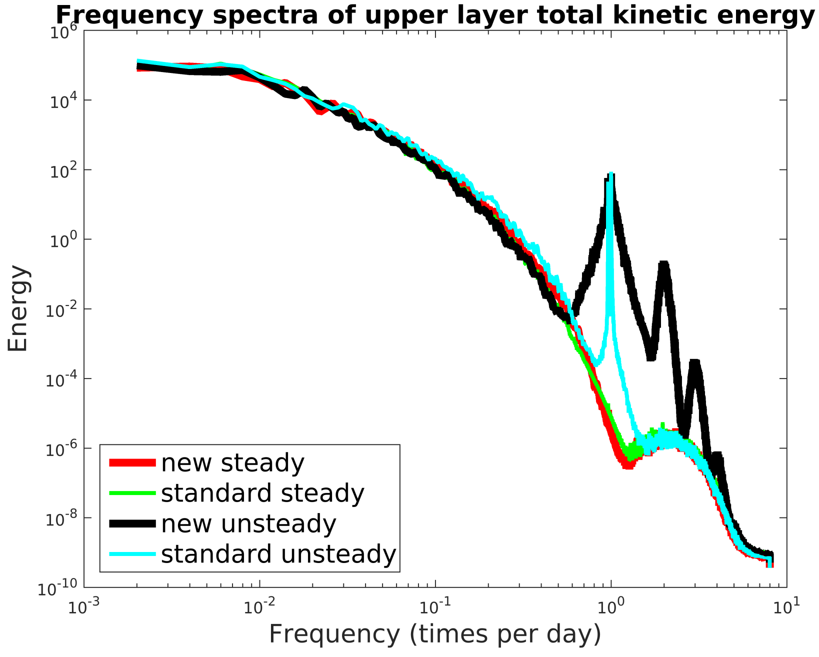


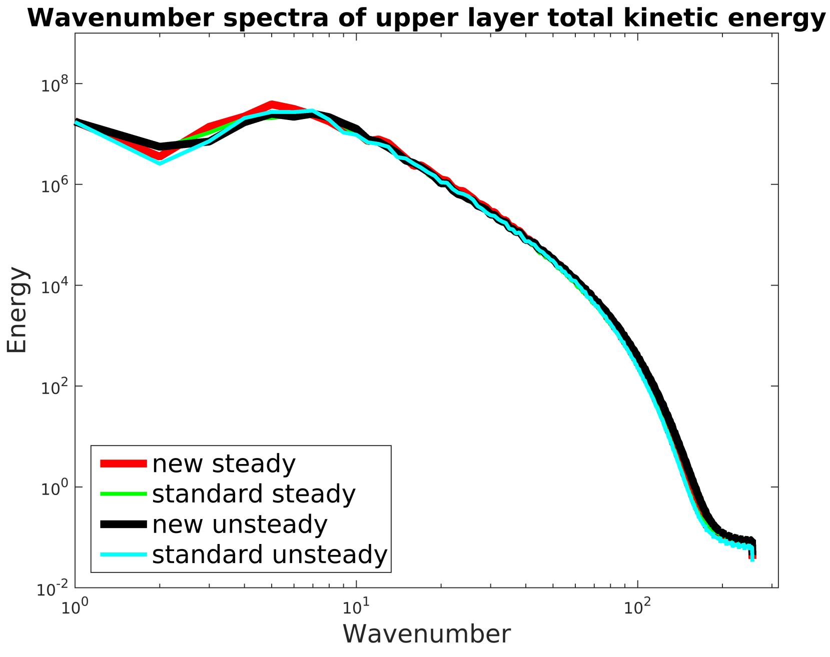
Film.8 Unsteady standard method



Comparison of different forcing formulations

Fig.5 Upper-layer total kinetic energy





Comparison of different forcing formulations

Fig.6 Upper-layer quasigeostrophic kinetic energy

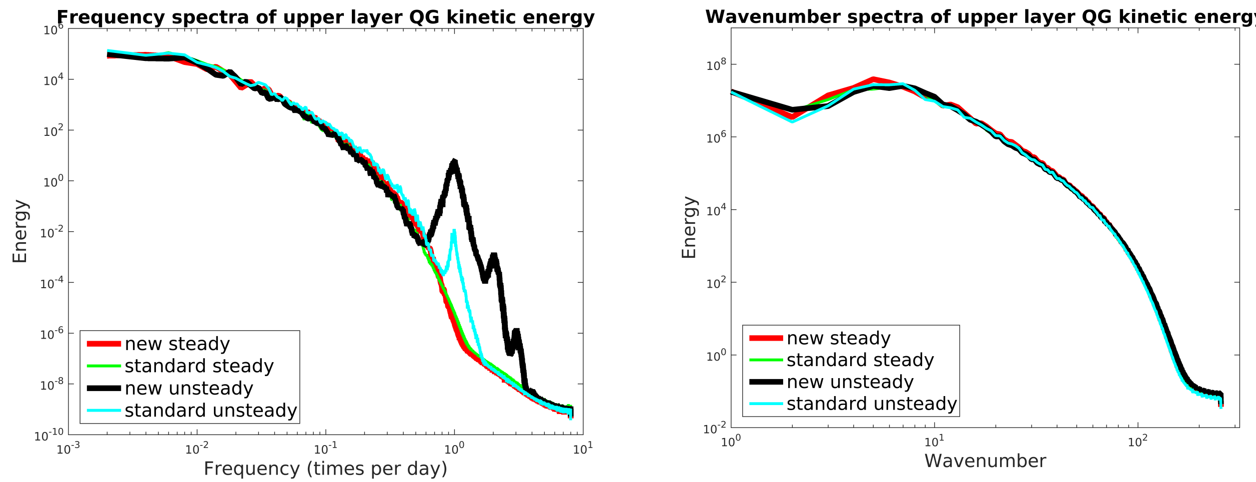
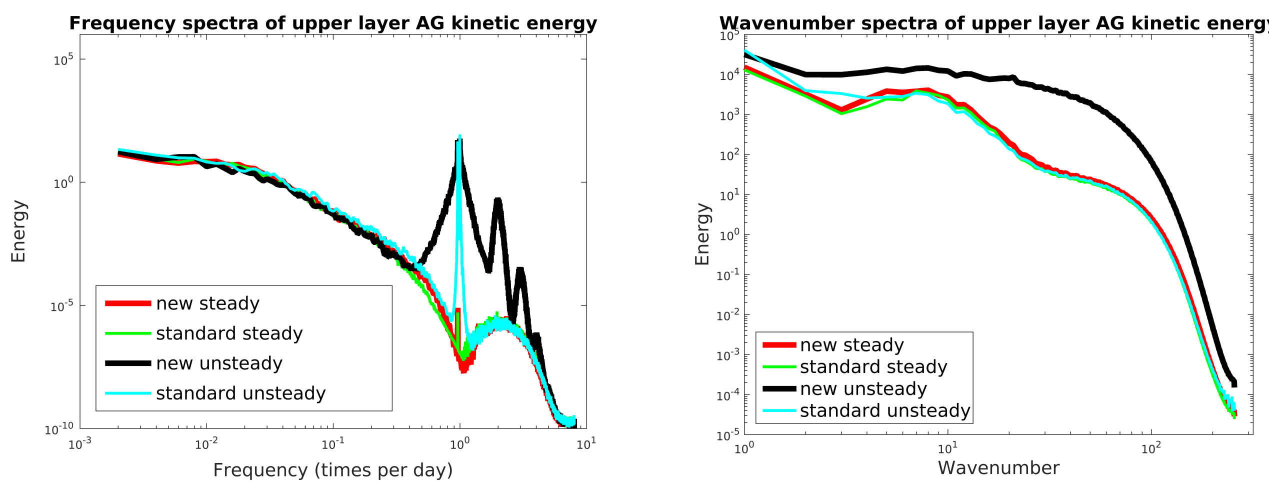


Fig.7 Upper-layer ageostrophic kinetic energy



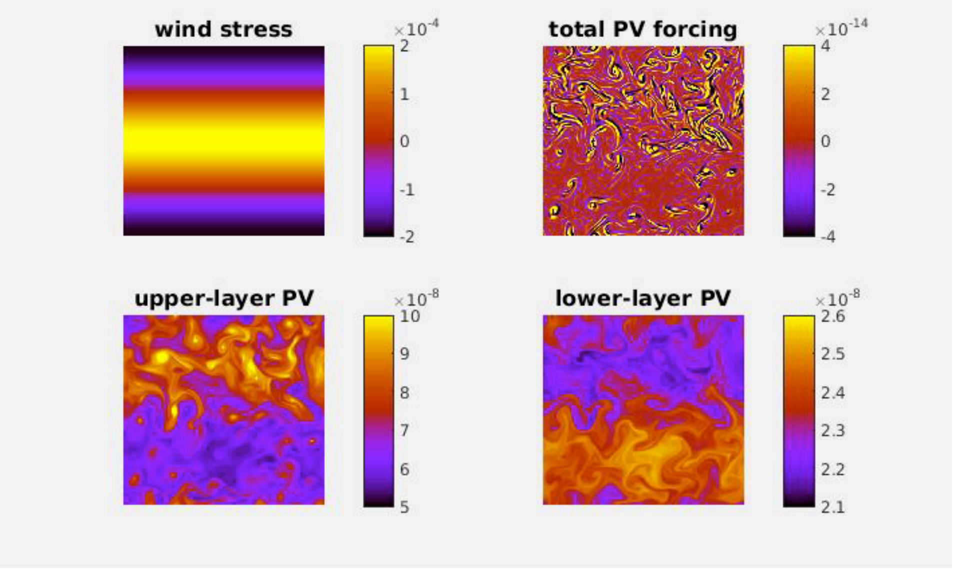
Both QG and AG kinetic energy parts are greatly enhanced at high frequencies, by the transition from standard formulation to new formulation.

Comparison of different forcing formulations: PV perspective

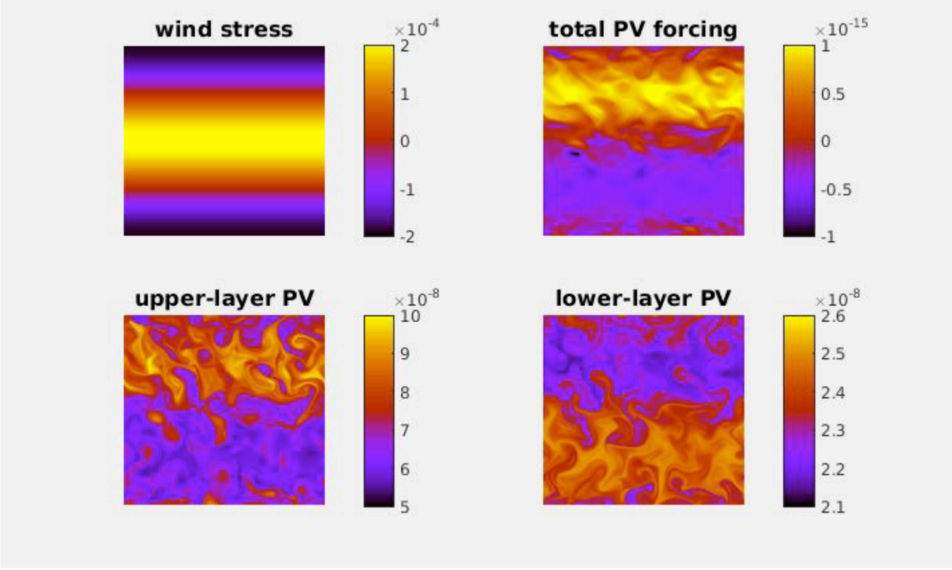
We can also analyze from the perspective of potential vorticity. First, we look at the RHS of the upper-layer PV equations, referred to as PV forcing.

Simulations	Standard method	New method
Upper-layer PV equations	$\frac{Dq_1}{Dt} = \frac{1}{h_1}(\nabla \times \frac{\vec{\tau}}{h_1})$	$\frac{Dq_1}{Dt} = \frac{q_1}{h_1} w_E$

Film.9 Unsteady new method



Film.10 Unsteady standard method



Comparison of different forcing formulations: PV perspective

Fig.8 Upper-layer PV forcing

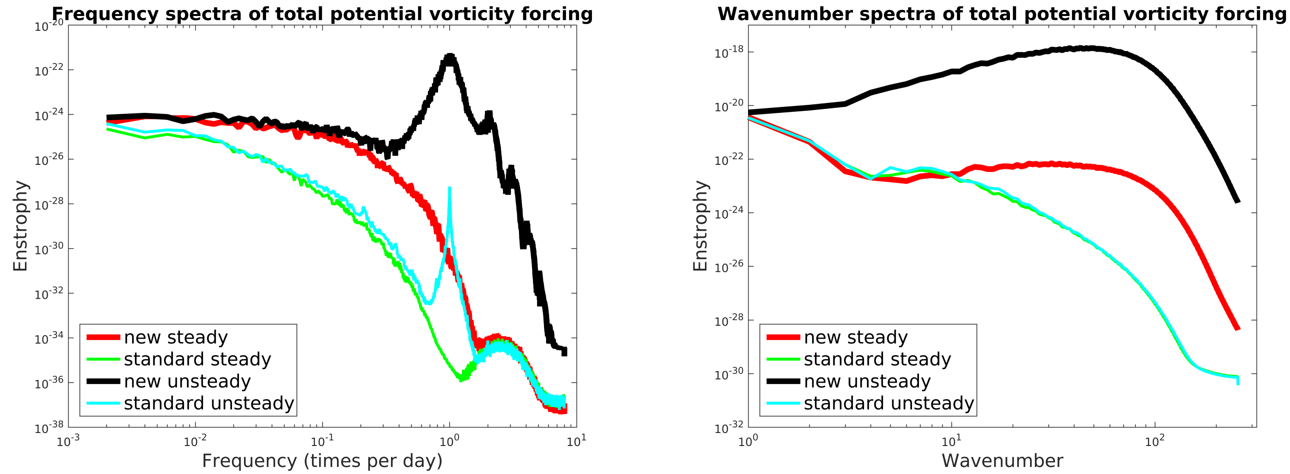
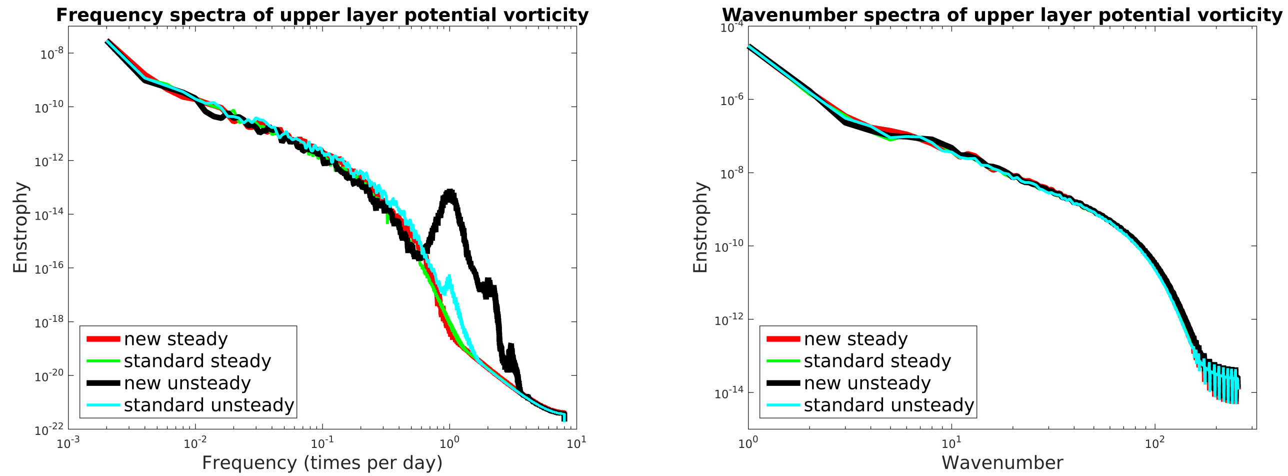


Fig.9 Upper-layer PV



Conclusion

- What is the impact of various Ekman formulations on the interior flow when the Ekman-layer is coupled to the interior?

  The high-frequency forcing excites near-inertial motion, and also the low-frequency nearly geostrophic part of the flow.
  Both QG and AG kinetic energy parts are greatly enhanced at high frequencies, by the transition from standard formulation to new formulation.

- Future work
  PV forcing vs PV.
  Need to include Ekman self-advection terms in Ekman equations.

Thanks for your attention.