Flow-dependent Ekman theory and its application to shallow water models

Yanxu Chen

Laboratoire de Météorologie Dynamique, École Normale Supérieure

(with David Straub and Louis-Philippe Nadeau)

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Air-sea momentum flux: modified by ocean **surface currents**

1. **Wind stress bulk formula**

$$\boldsymbol{\tau} = \rho_a c_d |\boldsymbol{u}_a - \boldsymbol{u}_o|(\boldsymbol{u}_a - \boldsymbol{u}_o)$$

Drag coefficient c_d also depends on ocean surface velocities.

2. **Ekman layer dynamics**

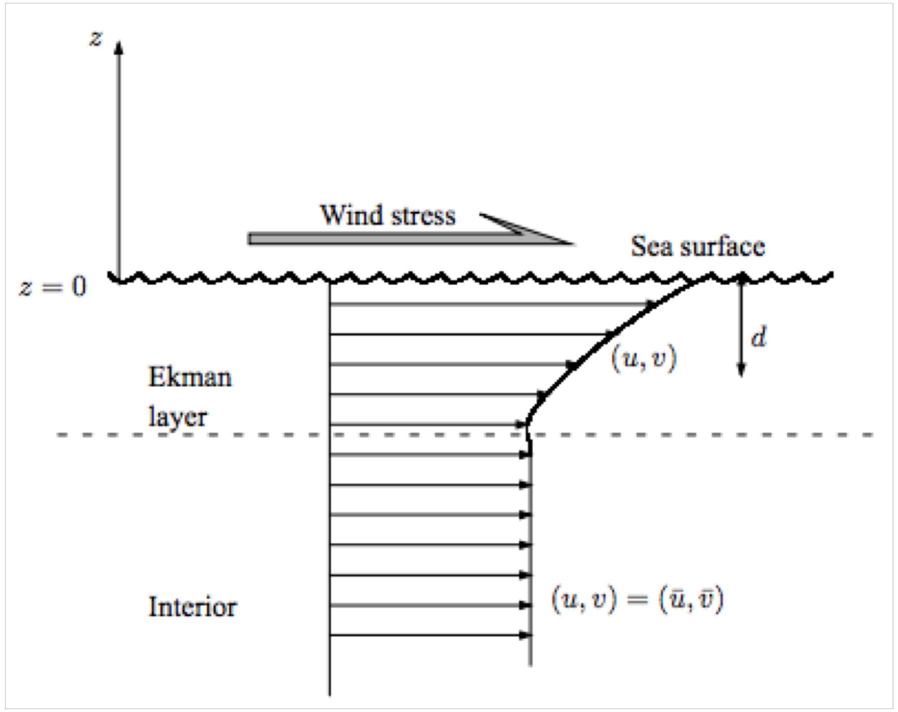
Classic Ekman-layer regime:

$$\begin{cases} f\hat{k}\times\boldsymbol{u} = \dfrac{1}{\rho_0}\nabla p + A_z\dfrac{\partial^2\boldsymbol{u}}{\partial z^2} \\ \boldsymbol{u} = \boldsymbol{u}_g + \boldsymbol{u}_{Ek} \end{cases}$$

$$f\hat{k}\times(\boldsymbol{u}_g + \boldsymbol{u}_{Ek}) = \frac{1}{\rho_0}\nabla p + \cancelto{0}{A_z\frac{\partial^2(\boldsymbol{u}_g)}{\partial z^2}} + A_z\frac{\partial^2(\boldsymbol{u}_{Ek})}{\partial z^2}$$

Ekman balance (surface layer)

Geostrophic balance
(interior ocean)



But observations depart significantly from this simple theory.

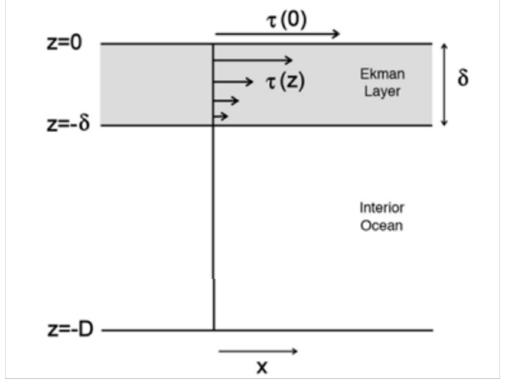
Development of the flow-dependent Ekman theory

People	Ekman (1905)	Stern and Niiler (1960s)	Wenegrat and Thomas (2017)
Content	Horizontal transport depends on the stress and Coriolis parameter f only.	Allows for shear of the surface velocity field to affect the transport.	Extends early results to better account for curvature in the surface flow path.
Ekman Transport	$U_{Ek} = \frac{\tau_y}{f}$ $V_{Ek} = -\frac{\tau_x}{f}$	$U_{Ek} \approx \frac{\tau_y}{f+\zeta}$ $V_{Ek} \approx -\frac{\tau_x}{f+\zeta}$	$R_0 \bar{u} \frac{\partial V_{Ek}}{\partial s} + (1 + R_0 2\Omega) U_{Ek} = \tau_n$ $R_0 \bar{u} \frac{\partial U_{Ek}}{\partial s} - (1 + R_0 \zeta) V_{Ek} = \tau_s$
Assumptions	Homogeneous deep stationary ocean.	Valid for plane parallel flows (e.g., straight jets); however, not explicitly solved for flows with curvature.	Curvilinear flows, with $R_{oe} \ll 1$ and $R_o < 1$; however, not easily applicable to complicated flow fields.

Outline of the following content

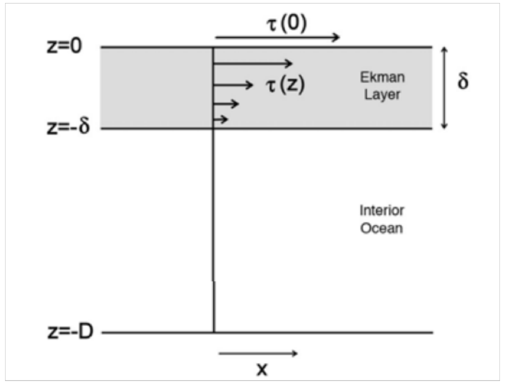
1. Flow-dependent Ekman formulation

2. What is the impact of various Ekman formulations on the Ekman transport for fixed wind stress and oceanic balanced vortex?



Steady wind stress at the surface;
Constant vortex in the ocean interior.

3. What is the impact of various Ekman formulations on the interior flow when the Ekman layer is coupled to the interior?



Seeking solutions for the coupling between Ekman layer and the interior.

Flow-dependent Ekman layer

$$\underbrace{\frac{\partial \boldsymbol{u}_{Ek}}{\partial t} + (\boldsymbol{u}_{Ek} \cdot \nabla)\boldsymbol{u}_0}_{} + (\boldsymbol{u}_0 \cdot \nabla)\boldsymbol{u}_{Ek} + (\boldsymbol{u}_{Ek} \cdot \nabla)\boldsymbol{u}_{Ek} + f\hat{k}\times\boldsymbol{u}_{Ek} = \frac{\partial \boldsymbol{\tau}}{\partial z} - A_h \nabla^4 \boldsymbol{u}_{Ek} \ldots$$

advection 1 advection 2 nonlinear (advection 3)

1. The time-dependent term has been added for simplifying the calculation.

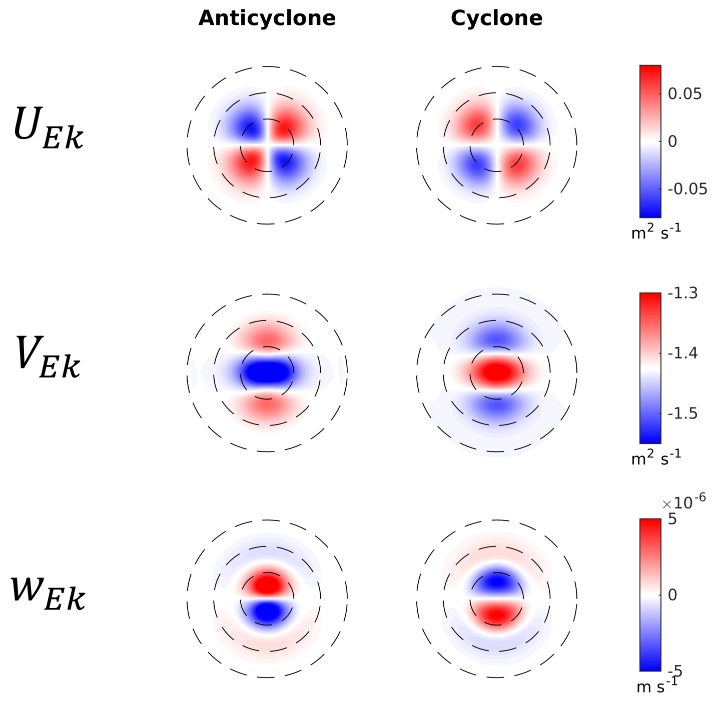
2. Scale analysis of the three advection terms: R_o, R_o^2 and $R_o \cdot R_{oe}$.
   (Assumptions used here: $R_{oe} \ll 1$ and $R_o < 1$.)

3. Advection 1 has been widely used (or added) to study wind forcing of near-inertial oscillations.

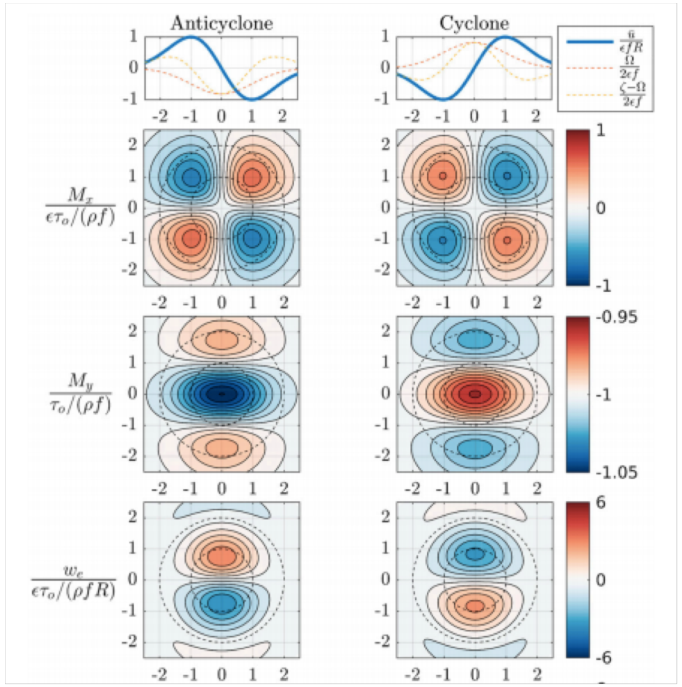
4. Vertical integration leads to the transport equation.

Section 1: the Ekman layer itself

Our Model Simulations



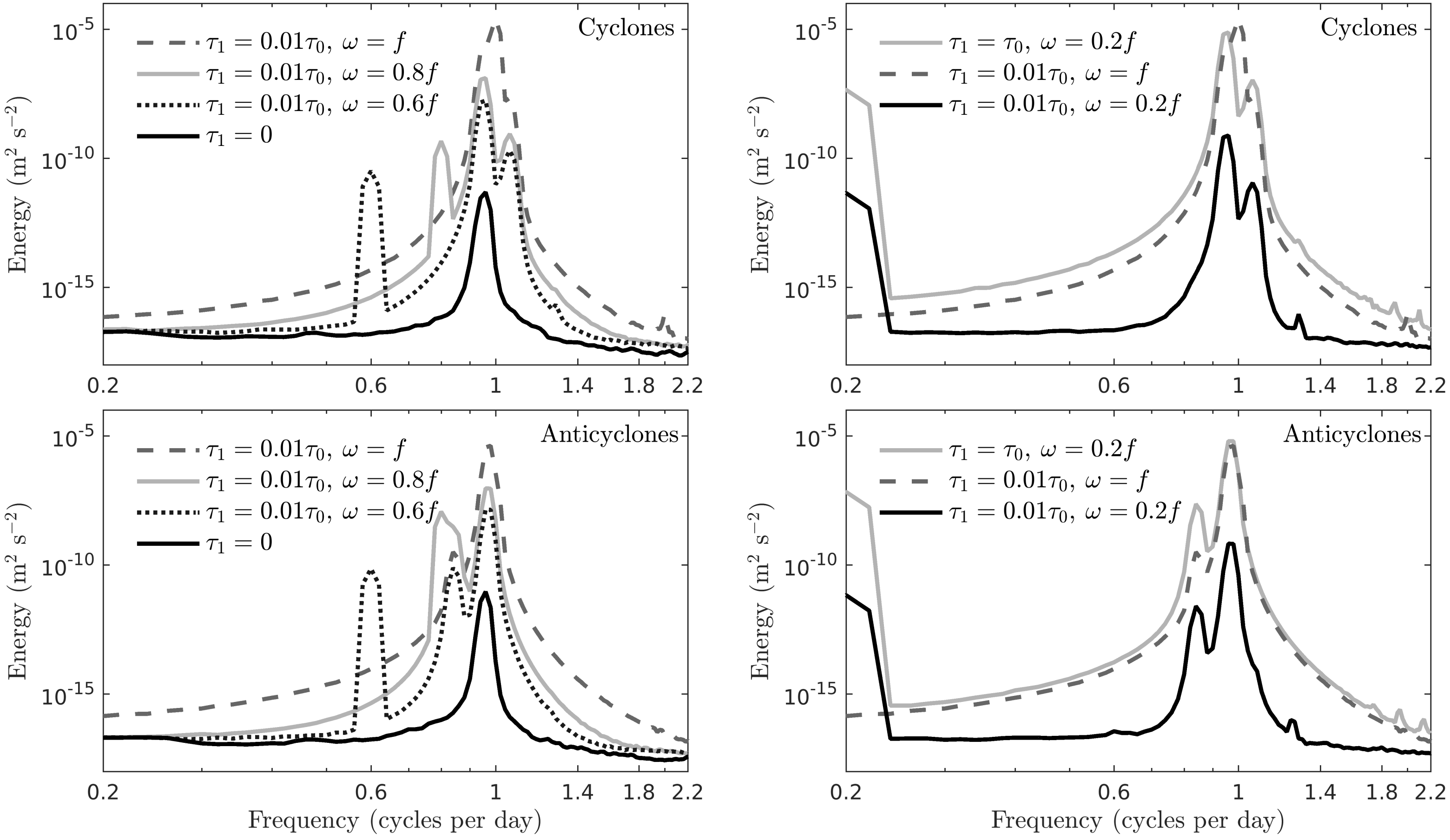
Wenegrat & Thomas simulations



The zonal transport develops a quadrupole pattern, emphasizing that the flow-dependent Ekman transport is not strictly perpendicular to the wind stress.

The meridional transport converges (diverges) on the north (south) side of the cyclonic vortex, with the pattern reversed for the vortex with anticyclonic flow.

Section 1: the Ekman layer itself

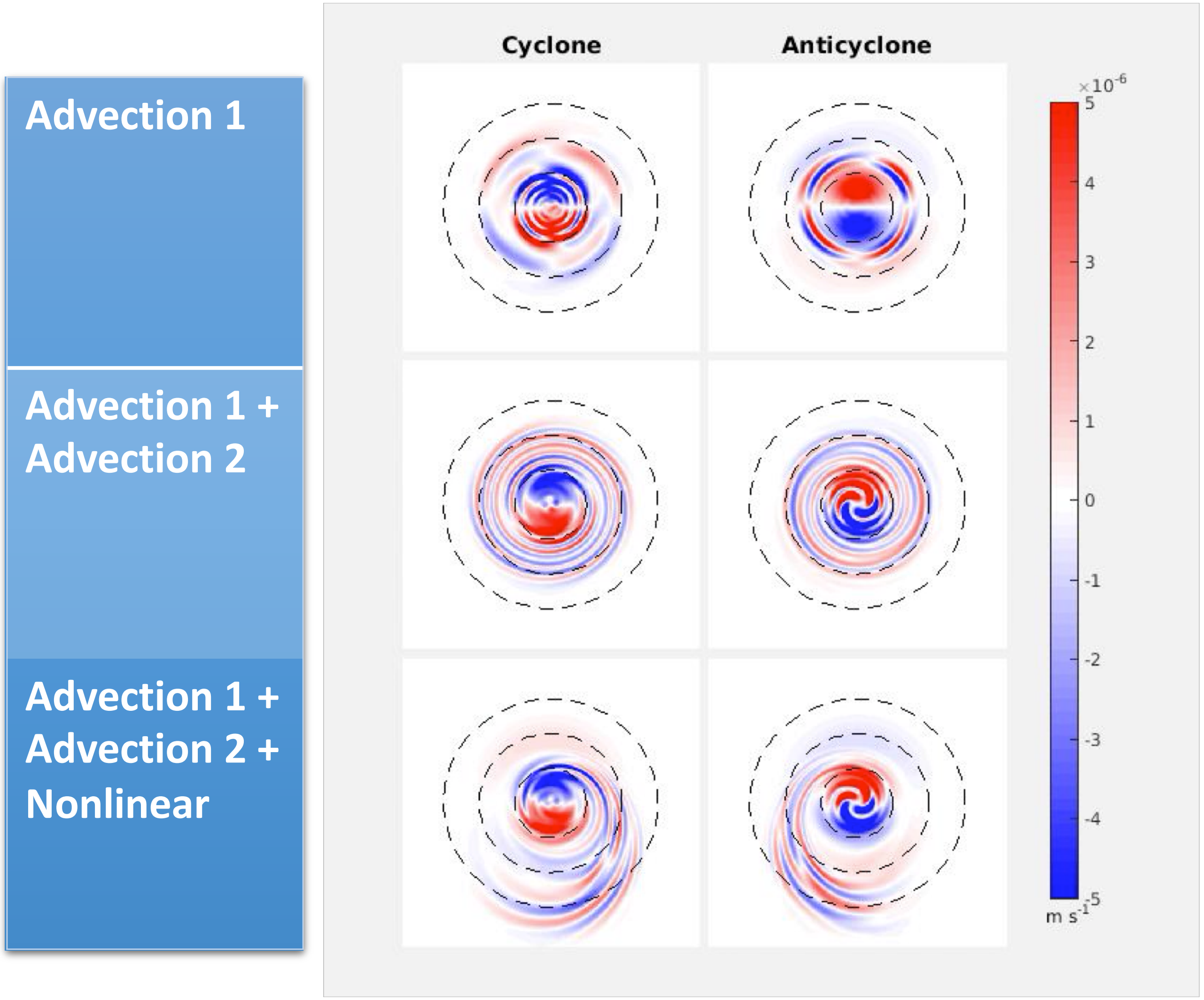


Frequency spectra of pumping velocities with forcing at different frequencies

$$\tau_{total} = \tau_0 + \tau_1 \cdot \sin(\omega t)$$

- High-frequency winds lead to responses at the same frequency, plus a component at f.
- Synoptic scale winds with large enough amplitude can be a forcing at f.

Section 1: the Ekman layer itself



Ekman pumping response with different regimes

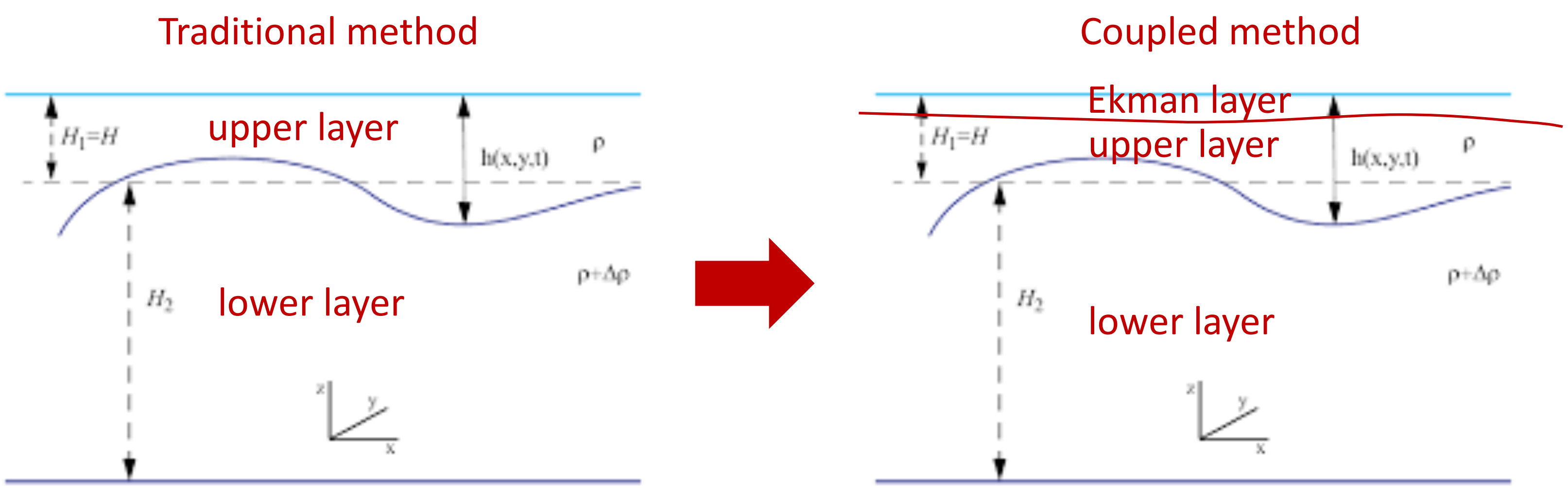
Recap:

- Ekman transport can include a component that is not perpendicular to the stress;
- The time dependence can introduce a near-inertial (high-frequency) component to the pumping velocities.

Section 2: Ekman-interior coupled model

Two different regimes

1. Wind stress is applied as a body force in the upper-layer momentum equation (traditional)
2. Use an explicit Ekman layer to force the upper-layer mass equation (coupled)



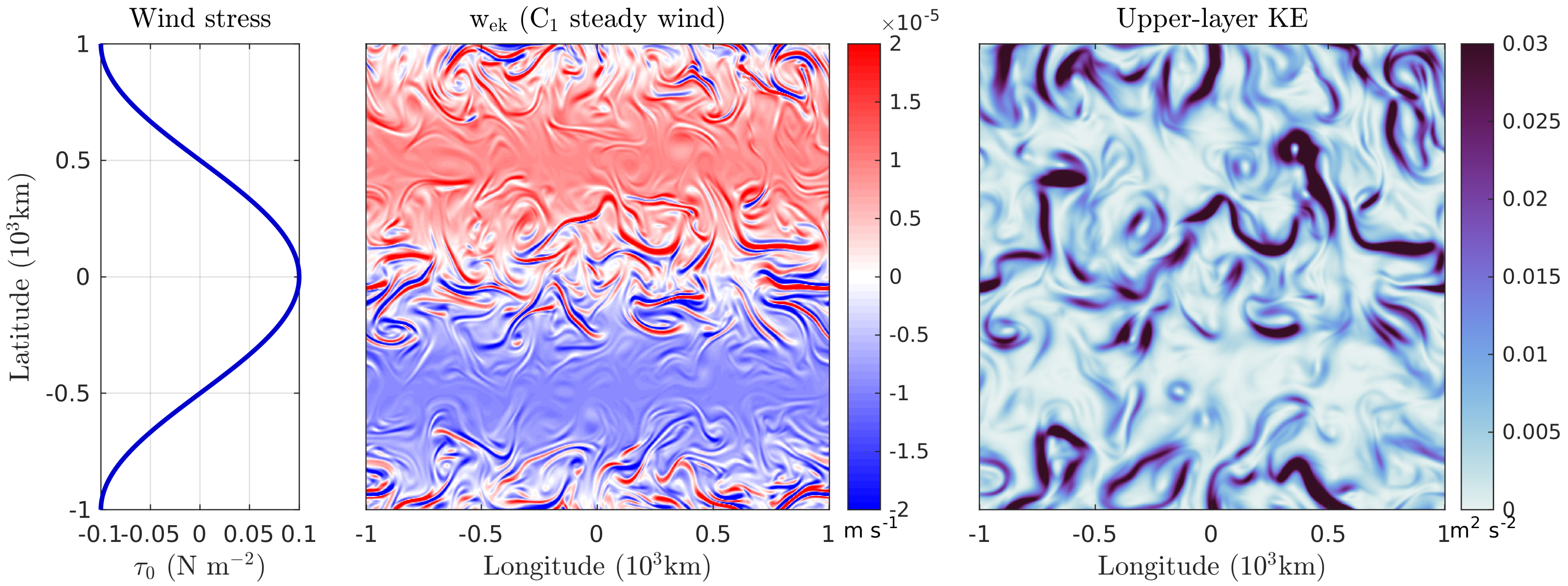
We consider a two-layer shallow water model with a slab Ekman layer in the top layer. Thus, we can use "Ekman pumping" as a forcing in the upper layer mass equation.

Model setup: two-layer rigid lid, domain size ($2000\text{km}\times 2000\text{km}$), resolution ($512$ grid points$\times$ 512 grid points), wind forcing τ is a cosine function of latitude.

Section 2: Ekman-interior coupled model

Simulations		Traditional method	Coupled method
Processes		Wind forcing → upper layer	Wind forcing → modified Ekman layer → upper layer
Equations	Ekman layer		Many options as described in model formulation (C1 model: advection1+2; C2 model: advection1+2+3)
	Upper-layer momentum	$\frac{\partial}{\partial t}\vec{u}_1 + (\vec{u}_1 \cdot \nabla)\vec{u}_1 + f\hat{z}\times\vec{u}_1 = \frac{\vec{\tau}}{h_1} - A_h\nabla^4\vec{u}_1$	$\frac{\partial}{\partial t}\vec{u}_1 + (\vec{u}_1 \cdot \nabla)\vec{u}_1 + f\hat{z}\times\vec{u}_1 = -A_h\nabla^4\vec{u}_1$
	Upper-layer mass	$\frac{\partial}{\partial t}h_1 + \nabla \cdot (h_1\vec{u}_1) = 0$	$\frac{\partial}{\partial t}h_1 + \nabla \cdot (h_1\vec{u}_1) = -w_E$ $(w_E = \nabla \cdot (\vec{U}_E))$

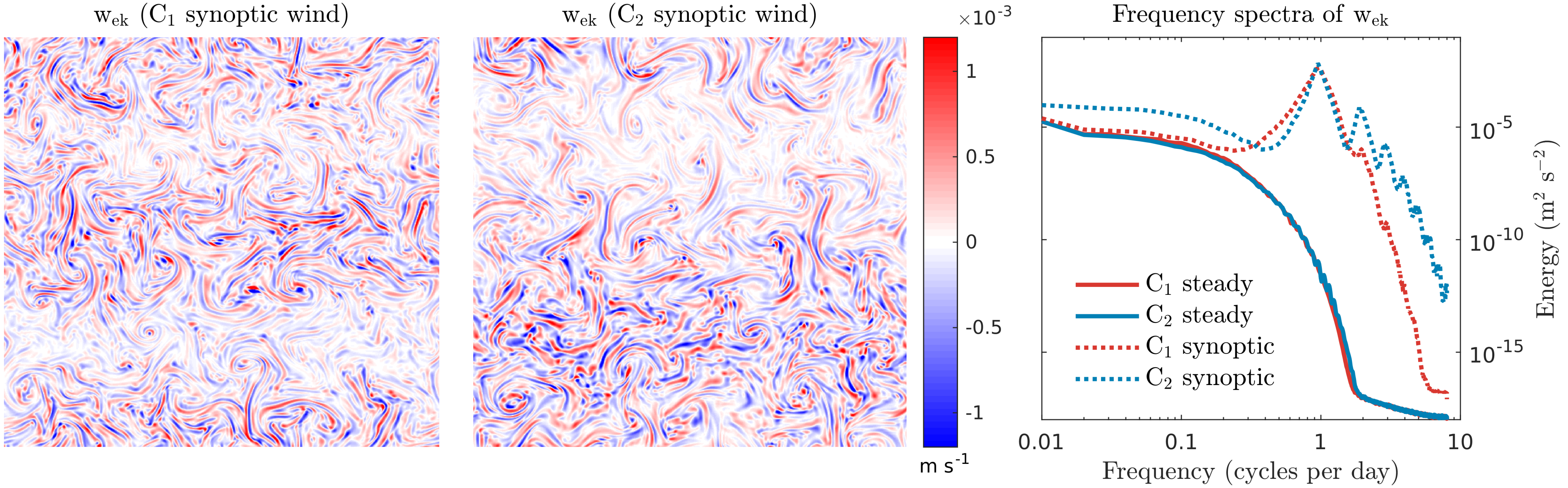
Section 2: Ekman-interior coupled model



C1 simulation with steady wind

(left: wind structure; mid: Ekman pumping; right: the upper-layer kinetic energy)

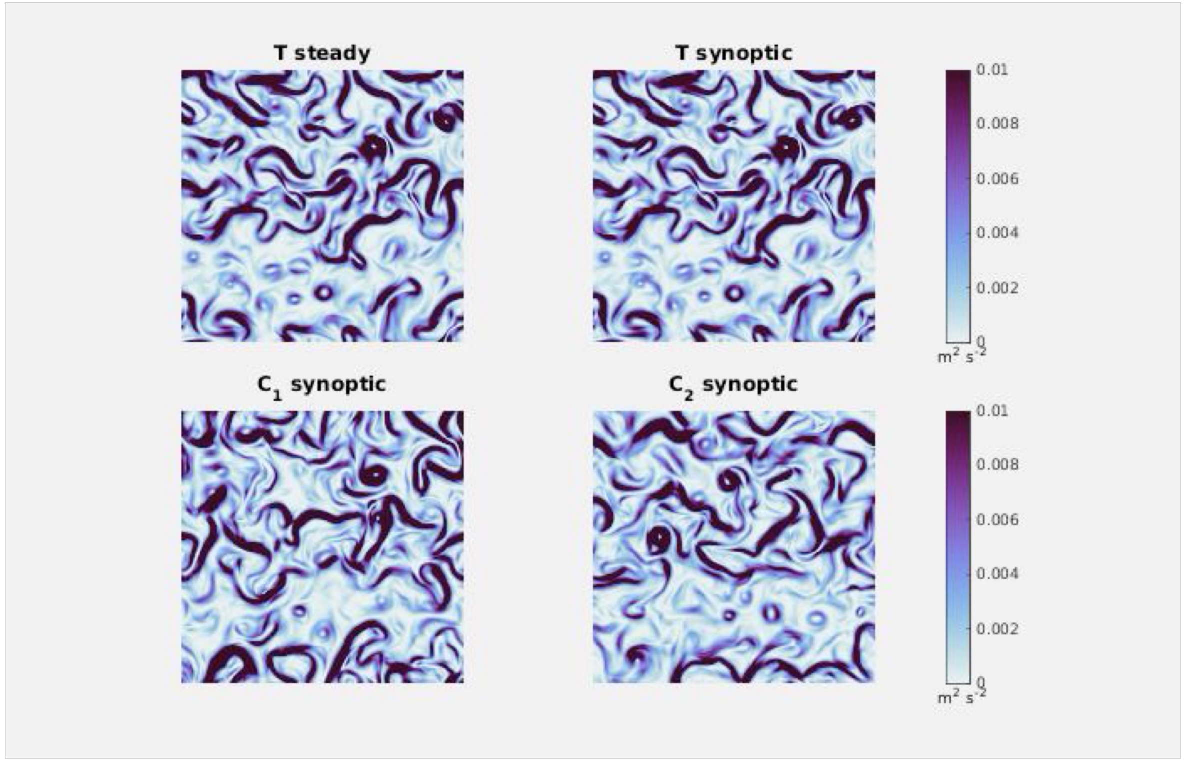
Section 2: Ekman-interior coupled model



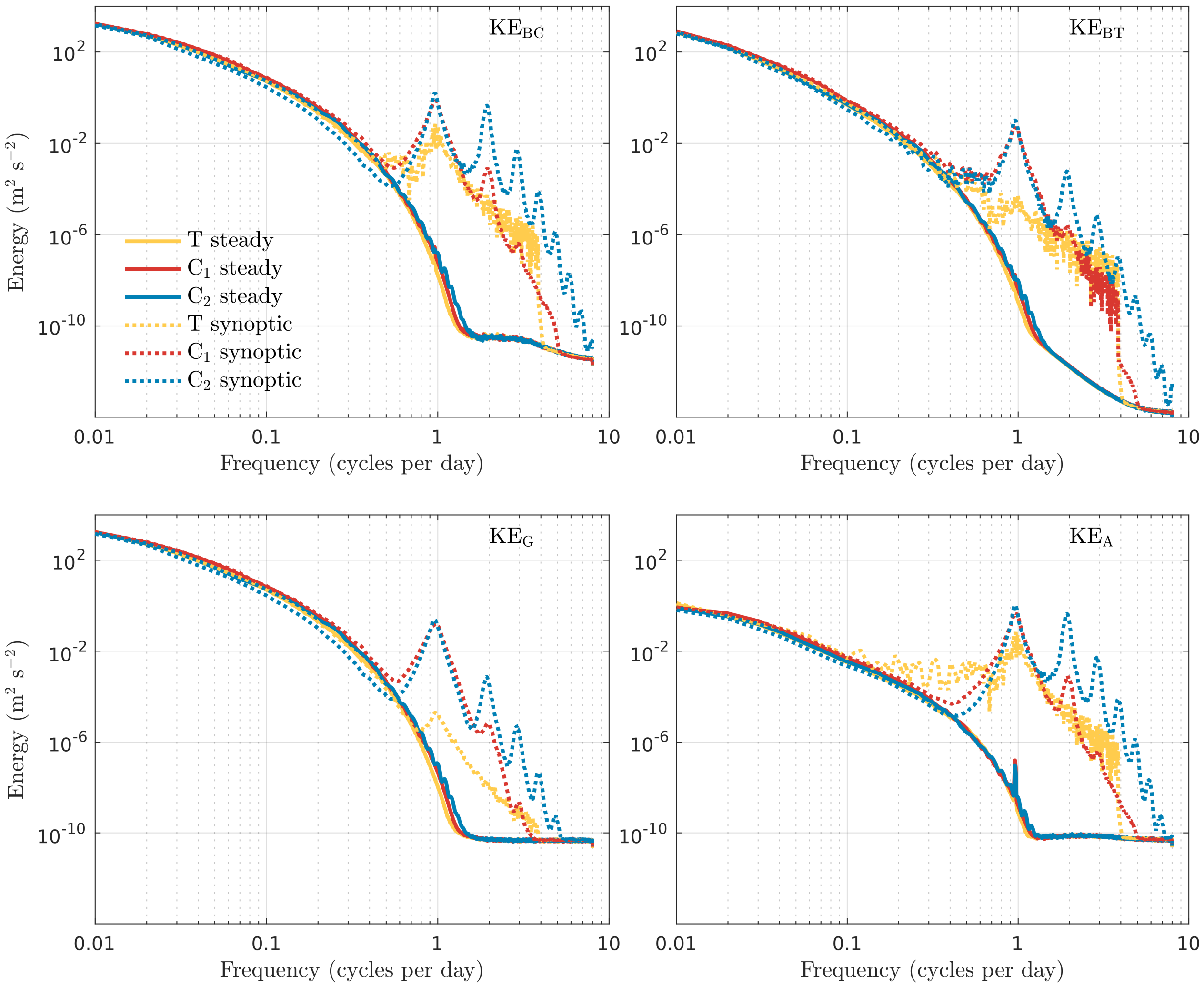
Ekman pumping

(left: C1 synoptic wind; mid: C2 synoptic wind; right: frequency spectra)

Section 2: Ekman-interior coupled model



Baroclinic KE of different simulations



Frequency spectra of KE response

Conclusions and discussions

- Flow-dependent Ekman layer can result in a transport that is not perpendicular to the wind.
- Synoptic wind can be a near-inertial forcing for the flow-dependent Ekman layer.
- With steady wind stress, the Ekman-interior coupled model is almost identical to the traditional two-layer shallow water model.
- For the coupled model, adding near-inertial components to the wind stress greatly enhances QG and AG kinetic energy response at high frequencies.

Thank you for your attention.