The Application of Flow-Dependent Ekman Transport to a Two-Layer Shallow Water Model

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Background

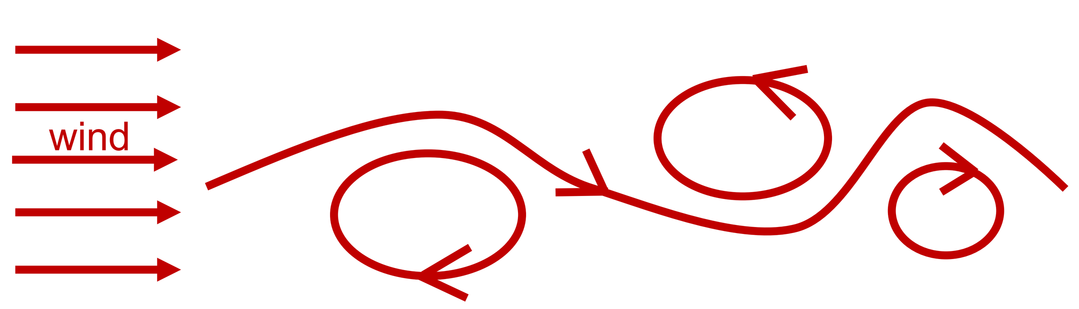
- Ekman transport and pumping are known to be modified by surface currents.

	Ekman (1905)	Stern (1965)	Wenegrat & Thomas (2017)
Content	Transport depends on the stress and the Coriolis parameter only.	Allows for shear in the surface velocity field to affect the transport: "nonlinear" Ekman theory.	Extends Stern's results to better account for curvature in the surface flow path.
Ekman transport	$U_E = \frac{\tau_y}{f}$ $V_E = -\frac{\tau_x}{f}$	$U_E = \frac{\tau_y}{f+\zeta}$ $V_E = -\frac{\tau_x}{f+\zeta}$	$\varepsilon\bar{u}\frac{\partial V_E}{\partial s} + (1+\varepsilon 2\Omega)U_E = \tau_n$ $\varepsilon\bar{u}\frac{\partial U_E}{\partial s} - (1+\varepsilon\zeta)V_E = \tau_s$
Assumptions	Homogeneous deep ocean at rest.	Valid for plane parallel flows (e.g., straight jets); however, validity for curvilinear flows has been questioned by Wenegrat & Thomas.	Curvilinear flows, with Ekman Rossby number <<1 and the balanced Rossby number <1.

- Note that W&T formulation has been carried out in curvilinear coordinates, thus, it would be difficult to apply their Ekman equations to complicated background flow fields, e.g., jets with a random shape, turbulent eddies, etc.
- We extend the W&T Ekman formulation by adding a time-dependent term. This step removes the need for integrating over streamlines, and also introduces a near-inertial component to the Ekman pumping.

Research Objective

- The questions are:
  Whether this flow-dependent Ekman layer produces a different structure for Ekman transport;
  Whether the interior flow responds to this Ekman layer differently from the one with usual boundary layer setup.



Model Formulation

- The modified Ekman layer

Ekman transport equations:

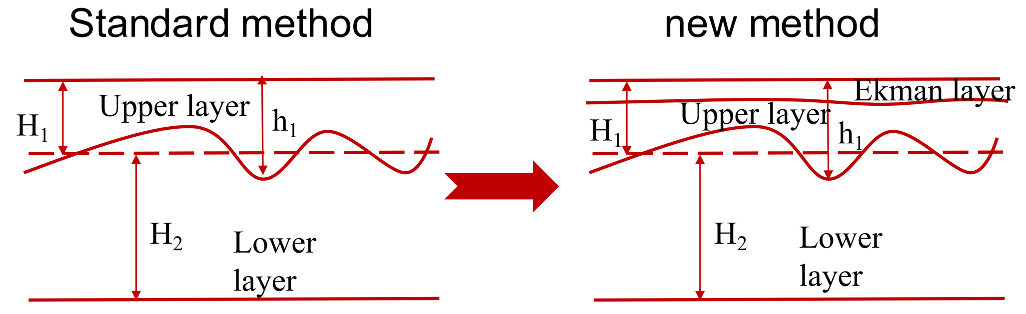
$$\frac{\partial}{\partial t}\vec{U}_E + (\vec{u}_0 \cdot \nabla)\vec{U}_E + (\vec{U}_E \cdot \nabla)\vec{u}_0 + f\hat{z}\times\vec{U}_E = \vec{\tau} - A_h\nabla^4\vec{U}_E$$

unsteady term, advection term, vorticity term, wind forcing term, diffusion term

Notice: $\vec{u}_0$ represents the background flow.

- The two-layer shallow water model

Typically, wind stress is applied as a body force over the ocean upper layer. We instead assume a thin Ekman-like layer embedded in the upper layer. Divergent Ekman transports then enter into the upper layer mass equation. We compare different formulations using a two-layer shallow water model.



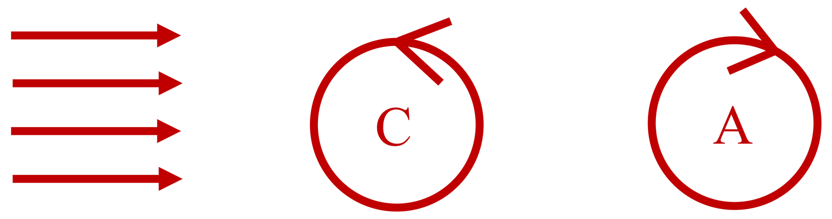
Simulation		Standard method
Process		Wind forcing → upper layer
Equations	Ekman transport	
	Upper-layer momentum	$\frac{\partial}{\partial t}\vec{u}_1 + (\vec{u}_1 \cdot \nabla)\vec{u}_1 + f\hat{z}\times\vec{u}_1 = \frac{\vec{\tau}}{h_1} - A_h\nabla^4\vec{u}_1$
	Upper-layer mass	$\frac{\partial}{\partial t}h_1 + \nabla\cdot(h_1\vec{u}_1) = 0$

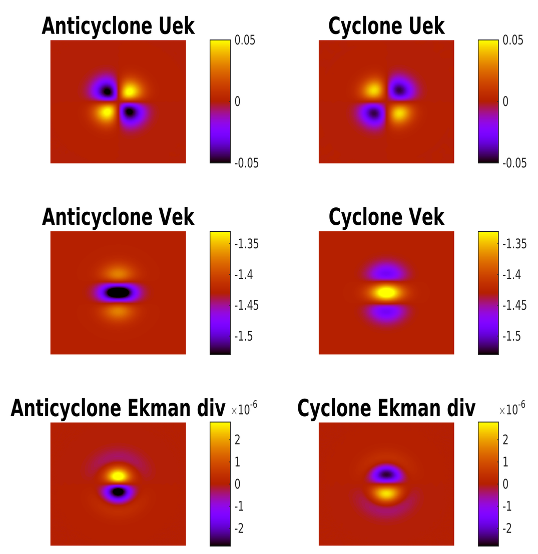
Simulation		New method
Process		Wind forcing → modified Ekman layer → upper layer
Equations	Ekman transport	$\frac{\partial}{\partial t}\vec{U}_E + (\vec{u}_1 \cdot \nabla)\vec{U}_E + (\vec{U}_E \cdot \nabla)\vec{u}_1 + f\hat{z}\times\vec{U}_E = \vec{\tau} - A_h\nabla^4\vec{U}_E$
	Upper-layer momentum	$\frac{\partial}{\partial t}\vec{u}_1 + (\vec{u}_1 \cdot \nabla)\vec{u}_1 + f\hat{z}\times\vec{u}_1 = -A_h\nabla^4\vec{u}_1$
	Upper-layer mass	$\frac{\partial}{\partial t}h_1 + \nabla\cdot(h_1\vec{u}_1) = -w_E$

Results

- Analysis 1: How is the Ekman transport of a balanced vortex different from intuition?

Here, we use a simple vortex to represent one eddy in the ocean and the vortex can be either cyclonic or anticyclonic.



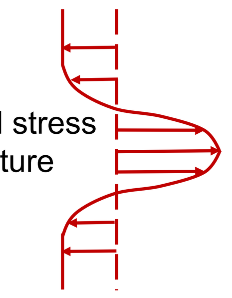


Scan here for the video of this simulation.

Our model produces transients (see videos), however, the time-average is consistent with Wenegrat and Thomas.

The zonal transport develops a quadrupole pattern, emphasizing that the nonlinear Ekman transport is not strictly perpendicular to the wind stress. The meridional transport converges (diverges) on the north (south) side of the cyclonic vortex, with the pattern reversed for the vortex with anticyclonic flow.

- Analysis 2: How does the interior respond to this modified Ekman layer?

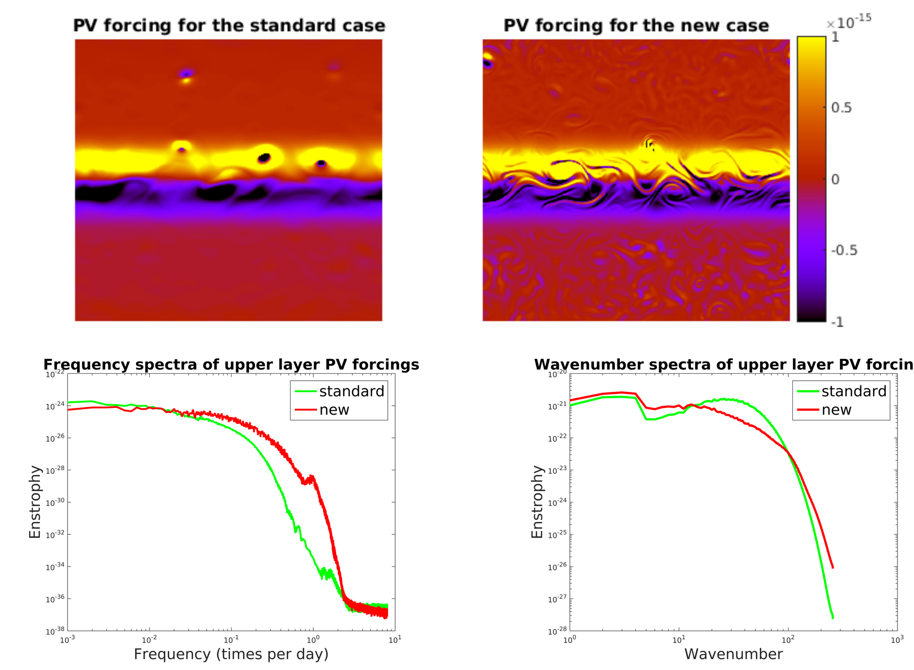


Scan here for videos of my simulations!

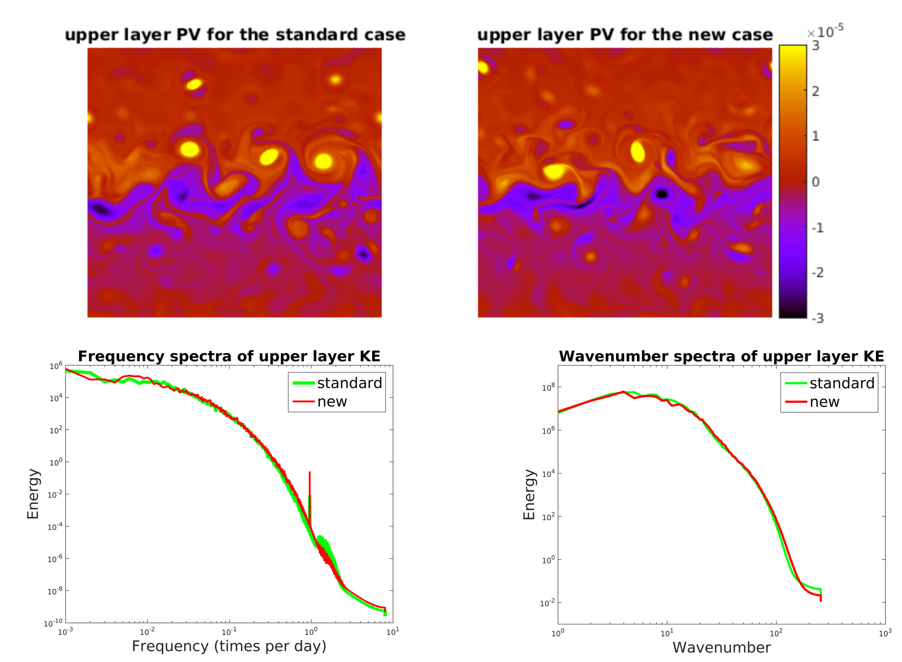
First, we analyze the RHS of the upper-layer PV equations, which can be called PV forcings. If PV forcings are different for standard and new Ekman representations, then we expect different interior responses.

Simulations	Standard method	New method
Upper-layer PV equations	$\frac{Dq_1}{Dt} = \frac{1}{h_1}(\nabla\times\frac{\vec{\tau}}{h_1})$	$\frac{Dq_1}{Dt} = \frac{q_1}{h_1}w_E$

Results



The new forcing shows more enstrophy input at high-frequencies, whereas the standard forcing shows a peak at intermediate-to-small scales. The latter appears related to coherent eddies with large interface height displacements.



Next, let's consider the upper-layer response. In contrast to notable differences in PV forcing, upper-layer kinetics of different simulations act similarly. Our future work will continue this analysis by adding a high-frequency component to the wind and compare upper-layer responses.

References

[1] Wenegrat and Thomas. Ekman transport in balanced currents with curvature.
[2] Niiler. On the Ekman divergence in an oceanic jet.
[3] Stern. Interaction of a uniform wind stress with a geostrophic vortex.