# A shallow water model forced by flow-dependent Ekman pumping

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## Background

Ekman transport and pumping are known to be modified by surface currents.

| | Ekman (1905) | Stern (1965) | Wenegrat & Thomas (2017) |
|---|---|---|---|
| Content | Transport depends on the stress and the Coriolis parameter only. | Allows for shear in the surface velocity field to affect the transport: "nonlinear" Ekman theory. | Extends Stern's results to better account for curvature in the surface flow path. |
| Ekman transport | $U_E = \frac{\tau_y}{f}$ <br> $V_E = -\frac{\tau_x}{f}$ | $U_E = \frac{\tau_y}{f+\zeta}$ <br> $V_E = -\frac{\tau_x}{f+\zeta}$ | $\varepsilon\bar{u}\frac{\partial V_E}{\partial s} + (1+\varepsilon 2\Omega)U_E = \tau_n$ <br> $\varepsilon\bar{u}\frac{\partial U_E}{\partial s} - (1+\varepsilon\zeta)V_E = \tau_s$ |
| Assumptions | Homogeneous deep ocean at rest. | Valid for plane parallel flows (e.g., straight jets); however, validity for curvilinear flows has been questioned by Wenegrat & Thomas. | Curvilinear flows, with Ekman Rossby number <<1 and the balanced Rossby number <1. |

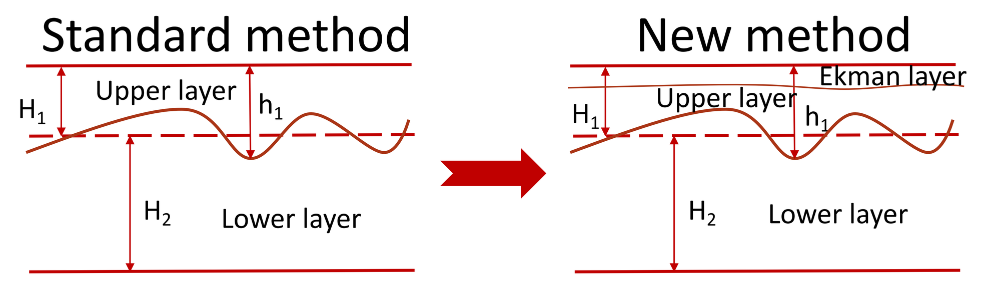
Note that W&T formulation has been carried out in curvilinear coordinates, thus, it would be difficult to apply their Ekman equations to complicated background flow fields, e.g., jets with a random shape, turbulent eddies, etc.

We extend the W&T Ekman formulation by adding a time-dependent term. This step removes the need for integrating over streamlines, and also introduces a near-inertial component to the Ekman pumping.

## Numerical Simulations

Typically, wind stress is applied as a body force over the ocean upper layer. We instead assume a thin Ekman-like layer embedded in the upper layer. Divergent Ekman transports then enter into the upper layer mass equation. We compare different formulations using a two-layer shallow water model.

| Simulations | | Standard method | New method |
|---|---|---|---|
| Processes | | Wind forcing → upper layer | Wind forcing → modified Ekman layer → upper layer |
| Equations | Ekman transport | | $\frac{\partial}{\partial t}\vec{U}_E + (\vec{u}_1\cdot\nabla)\vec{U}_E + (\vec{U}_E\cdot\nabla)\vec{u}_1 + f\hat{z}\times\vec{U}_E = \vec{\tau} - A_h\nabla^4\vec{U}_E$ |
| | Upper-layer momentum | $\frac{\partial}{\partial t}\vec{u}_1 + (\vec{u}_1\cdot\nabla)\vec{u}_1 + f\hat{z}\times\vec{u}_1 = \frac{\vec{\tau}}{h_1} - A_h\nabla^4\vec{u}_1$ | $\frac{\partial}{\partial t}\vec{u}_1 + (\vec{u}_1\cdot\nabla)\vec{u}_1 + f\hat{z}\times\vec{u}_1 = -A_h\nabla^4\vec{u}_1$ |
| | Upper-layer mass | $\frac{\partial}{\partial t}h_1 + \nabla\cdot(h_1\vec{u}_1) = 0$ | $\frac{\partial}{\partial t}h_1 + \nabla\cdot(h_1\vec{u}_1) = -w_E$ |




We also consider other simple formulations, e.g., using $\frac{\vec{\tau}}{H_1}$ as a body force or the classic Ekman pumping as a forcing for the upper-layer mass equation. However, only the "standard" and "new" methods are considered here.

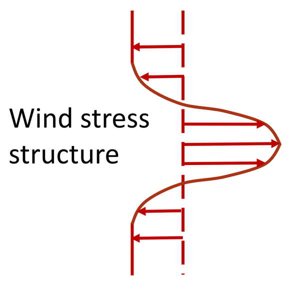
What factors impact the response of ocean interior flow to surface wind stress?

1. Representation of the Ekman layer
2. Interaction between eddies and the Ekman layer

Wind Stress → Ekman layer → Ocean Interior Flow

We introduce a time-dependent Ekman layer which interacts with eddy velocities. This new representation of the Ekman layer benefits associated dynamical processes.

## Analysis: Steady Wind Stress with Shear

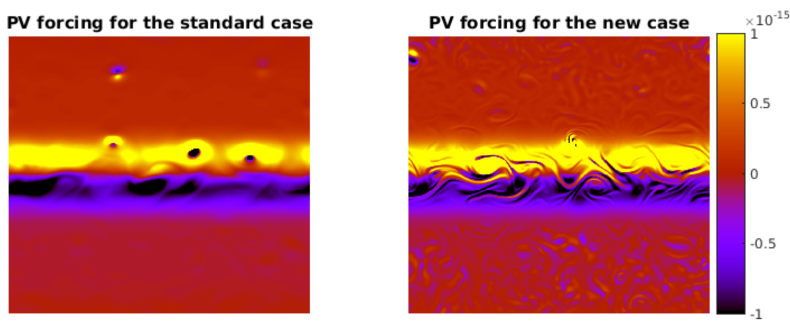


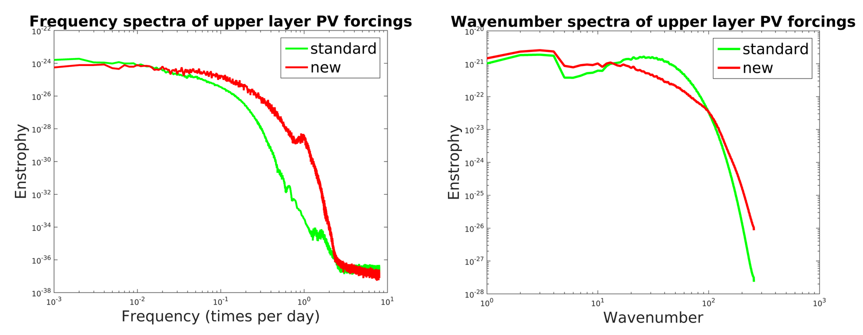

Scan here for videos of my simulations!!!

➢ First, let's look at the forcings. We focus on PV forcing to get an "apples-to-apples" comparison.

Here, we analyze the RHS of the upper-layer PV equations, which can be called PV forcings.

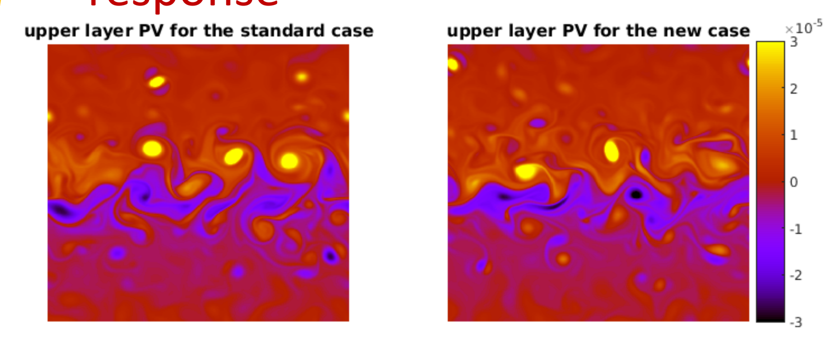
| Simulations | Standard method | New method |
|---|---|---|
| Upper-layer PV equations | $\frac{Dq_1}{Dt} = \frac{1}{h_1}(\nabla\times\frac{\vec{\tau}}{h_1})$ | $\frac{Dq_1}{Dt} = \frac{q_1}{h_1}w_E$ |

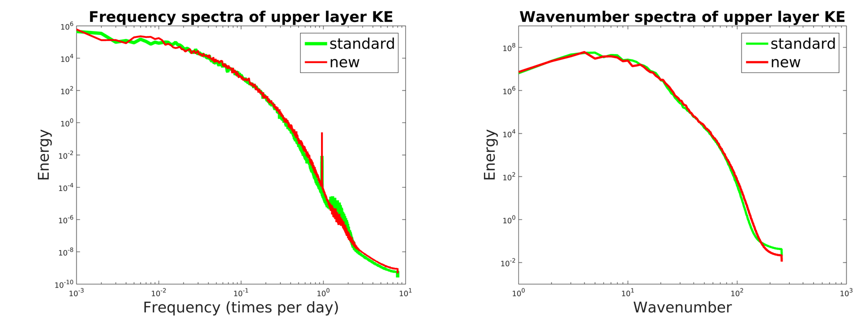



The new forcing shows more enstrophy input at high-frequencies, whereas the standard forcing shows a peak at intermediate-to-small scales. The latter appears related to coherent eddies with large interface height displacements.

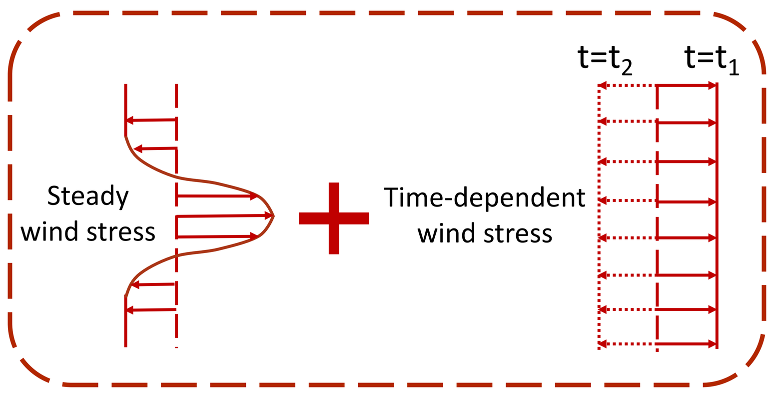
➢ Next, let's consider the upper-layer response

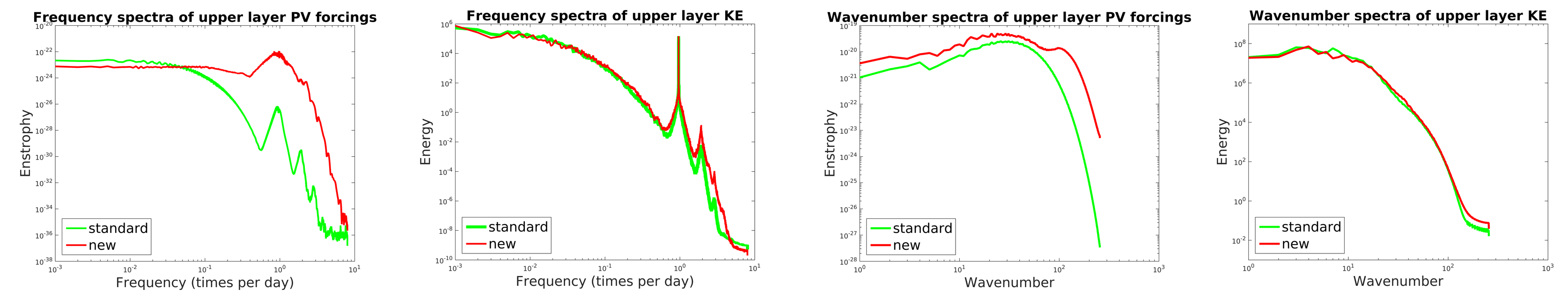



In contrast to notable differences in PV forcing, upper-layer kinetics of different simulations act similarly.

We next add a high-frequency component to the wind, which oscillates at Coriolis frequency. Again, large differences are evident in the (PV) forcing fields, but these do not lead to large differences in the response. Our future work asks why.




## References

[1] Wenegrat and Thomas. Ekman transport in balanced currents with curvature.
[2] Niiler. On the Ekman divergence in an oceanic jet.
[3] Stern. Interaction of a uniform wind stress with a geostrophic vortex.

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